Inter- and Intragenerational Distribution and the Valuation of Natural Capital

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Abstract: This paper studies how the intra- and intergenerational distribution of income and wealth affect the economic valuation of environmental public goods derived from natural capital. We consider both a single payment or a constant payment fraction share over time and the willingness to pay (WTP) for a marginal change of the level or the growth rate of the environmental public good. We find that the intragenerational distribution affects the intertemporal valuation of environmental goods derived from natural capital. We show that for both payment vehicles, societal mean WTP for the level as well as the growth rate of natural capital decreases (increases) with intratemporal income inequality if environmental goods derived from natural capital and consumption goods are substitutes (complements). We obtain closed-form adjustment factors for benefit transfer to control for differences in dynamic aspects between study and policy sites, such as income growth, the growth rate of the environmental goods, and interest rates. Our results are relevant for the economic appraisal of environmental policy as well as natural capital accounting and management.

JEL-Classification: D63, H43, Q51

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1 Introduction

Sustainability economics is concerned with a just distribution of economic resources within and across generations, and efficiency in the attainment of these normative goals (Baumgärtner and Quaas 2010). The economic valuation of environmental goods and natural capital has gained considerable interest both in science (Kinzig et al. 2011, Fenichel and Abbott 2014, Pascual et al. 2017) and policy advice (Inclusive Wealth Project 2016, United Nations et al. 2014). Likewise, recent years have shown an increased interest in the effects of economic inequality that has been rising in many countries around the world (Piketty 2014, Alvaredo et al. 2017, 2018).

Economic efficiency requires that non-market environmental goods that have public good characteristics are supplied to the extent that the aggregate willingness to pay (WTP), that is the sum of household WTPs, equals the marginal (opportunity) cost of supplying environmental goods (Lindahl 1928, Samuelson 1954). This requires determining society’s aggregate WTP, which in general depends on the distribution of income and wealth within and across generations. Yet, the literature on non-market valuation does not explicitly consider the distribution of income or wealth with very few exceptions (Drupp et al. 2018b, Kriström and Riera 1996).

We study how the intra- and intertemporal distribution of private income (or wealth) affect the economic valuation of environmental goods derived from natural capital that exhibit characteristics of public goods. We thereby extend upon the static setting in the recent literature on how the intra-temporal distribution of income affects the valuation of environmental public goods (Ebert 2003, Baumgärtner et al. 2017). To examine the effect of economic inequality on societal valuation of environmental goods, this literature employs a stylized modelling framework that abstracts from how natural capital translates into ecosystem services or environmental goods. Specifically, Baumgärtner et al. (2017) study a setting in which a household has constant-elasticity-of-substitution (CES) preferences concerning a market-traded consumption good and a non-market traded environmental public good. In this setting, the CES directly determines the constant income elasticity of WTP. For households that have identical preferences but
differ in exogenously given income approximated with a log-normal distribution, they find that mean societal WTP for environmental public goods decreases (increases) with income inequality if and only if the environmental public good and manufactured goods are substitutes (complements). Thus the degree of substitutability of environmental public goods vis-a-vis market-traded goods is the key determinant of how the benefits derived from environmental public goods are distributed.

Since a core aspect of sustainability concerns distributional issues over time and how scarce natural resources and services can be managed to the benefit of future generations, an analysis of how the intra- and intertemporal distribution of income affects mean WTP is lacking for a comprehensive valuation from a sustainability perspective (Drupp et al. 2018b). This is important not least because currently living societies value environmental goods that derive from a stock of natural capital and evolve over time—for example, the existence of evolving species or climate stability, just to name a few. Extending the analysis of how the intra- and intertemporal distribution affects society’s intertemporal WTP for environmental goods relates to recent work in the literature on social discounting. For example, Emmerling (2018), Fleurbaey and Zuber (2015) and Gollier (2015) study inter- and intra-generational distribution in the context of discounting of a single consumption good. As far as the intertemporal distribution of market-traded and non-market-traded goods is concerned, our paper is related to the literature on dual discounting and relative price changes (e.g. Baumgärtner et al. 2015, Drupp 2018, Gollier 2010, Traeger 2011, Weikard and Zhu 2005). The change in relative prices of non-market environmental goods is determined by their degree of substitutability vis-a-vis market goods as well as the difference in their good specific growth rates. These determinants will also feature prominently in our analysis.

We generalize the static model of Baumgärtner et al. (2017) to an intertemporal setting. To capture the intertemporal dimension in a way that allows for closed-form solutions, we make the following limiting assumptions. First, we consider a simple mapping of natural capital to the environmental goods and services it provides. We thereby focus on analyzing non-consumptive environmental services, such as the existence value of biodiversity. It is in particular for these non-use services that WTP information is
crucial for public policy. For an analysis of wealth reallocation due to climate change of provisioning services derived from natural capital, such as the fishery, that features more complex natural capital dynamics see Fenichel et al. (2016); Second, we consider specific exogenously given time paths of consumption or income and the provision of environmental goods. We thus abstract from savings and optimal management and follow the approach that Arrow et al. (2003) have taken for computing shadow prices in non-optimal economies to determine the household and aggregate WTP for environmental goods for a given ‘resource allocation mechanism’. In particular, we study the case of exponential growth or decline as a special case. However, we also show how this setting of exponential growth of income can be derived from an endogenous growth model; Third, we assume that income and consumption are log-normally distributed in each period. This implies that given positive growth, absolute income inequality will increase over time while relative income inequality will remain constant; Fourth, in extending the instantaneous CES utility function to a dynamic setting, we assume—following recent theory on intertemporal decision-making (e.g. Golosov et al. 2014, Quaas and Bröcker 2016)—that there is a specific relationship between the elasticity of substitution and the intertemporal elasticity of substitution with respect to the aggregate consumption bundle. This allows deriving a closed-form intertemporal utility function under reasonable conditions on the relationship between growth rates, discount rates and the elasticity of substitution. Given this set-up, we consider compensating surplus as WTP for two different payment schemes—a single payment in the initial period as well as a constant payment fraction paid at each point in time—and for two different marginal changes to the provision of the environmental public good—a change in the initial level as well as in the growth rate of the environmental good.

We confirm key results from the static analysis in this more general dynamic setting and derive additional results regarding the effect of intergenerational distribution on natural capital valuation. We show that societal WTP as single or constant payment fractions elicited for levels or growth rates of the environmental goods increases with initial mean income, and decreases (increases) with initial relative income inequality if and only if the environmental good and the manufactured are substitutes (complements).
In addition, we show that societal WTP elicited for the level of the environmental goods as a constant payment fraction increases with income growth for complements or the Cobb-Douglas case, but that it is possible that societal WTP declines in the case of substitutes. Furthermore, we show that societal WTP elicited for the level of the environmental goods as a single payment or a constant payment fraction increases (decreases) with the growth rate of environmental goods if and only if environmental goods are a substitute (complement) to manufactured goods. Finally, we derive transfer factors for value or benefit transfer to account for differences in the distribution of income, income growth, growth of the environmental good, interest rates and other characteristic between a study and a policy site. We illustrate and quantify the effect sizes of our results for a case study on the intertemporal valuation of non-consumptive environmental public goods derived from global biodiversity.

Our results are relevant in several respects. First, incomes of those who benefit from natural capital are highly unequal. A proper valuation of natural capital requires to adequately take the effects of inequality into account. Failing to do so may lead to invalid figures, e.g. for natural capital accounting. Our conceptual model provides a guideline how practical studies can proceed to adequately take the distribution of income and wealth into account in natural capital valuation. Second, the distributional consequences of conservation, or degradation, of natural capital are crucial for the political economy of the decision on such measures. From the results of our analysis one can infer whether protection or degradation of natural capital has regressive or progressive effects. Finally, our results add to the emerging of structural benefit transfer. We derive novel closed-form adjustment factors for benefit transfer to control for differences in dynamic aspects between study and policy sites, such as income growth, the growth rate of the environmental goods, and interest rates. This is in particular needed when valuing ecosystem services derived from natural capital.

The remainder of the paper is structured as follows. We present the model in Section 2, our valuation concepts in Section 3 and results in Section 4. We apply our findings empirically for global biodiversity conservation in Section 5. We discuss limitations in Section 6 and conclude in Section 7. The Appendix contains all proofs.
2 Model

We generalize the static model of Baumgärtner et al. (2017) to an intertemporal setting. A society $s$ consists of a population of $n$ households, labelled $i = 1, \ldots, n$, who derive utility from the consumption of two composite goods—a market-traded private consumption good $C_i$ and an environmental good $E_i$. The environmental good derives from a stock of natural capital $N$ in the form of instantaneous dividends or services, with $E = \psi \times N$, where $\psi$ maps the stock of natural capital into environmental goods provided at each point in time. While this mapping is admittedly simple, it is a useful approximation among others for non-consumptive environmental services derived from natural capital, such as the existence value of biodiversity. Furthermore, all households consume the environmental good at the same fixed level, i.e. $E_i = E$. We therefore consider a pure public good. Individuals have identical preferences over the consumption good and the environmental public good, represented by the instantaneous constant-elasticity-of-substitution (CES) utility function

$$u^i(C_i, E) = \left(\alpha C_i^{\theta \frac{\theta - 1}{\theta}} + (1 - \alpha) E^{\theta \frac{\theta - 1}{\theta}}\right)^{\frac{\theta}{\theta - 1}},$$

where $\theta$, with $0 < \theta < +\infty$, is the CES between the two goods, and $0 < \alpha < 1$ is a share parameter determining the initial weights of consumption goods in utility. The CES function contains the special cases where the consumption good and the environmental good are substitutes ($\theta > 1$), Cobb-Douglas ($\theta = 1$) and perfect complements ($\theta < 1$).

To focus the model on the task of valuation only, both goods evolve over time $t$ and their time path is exogenously given, with $t = 0, \ldots, T$. This implies that we are not concerned here about the optimal management of natural capital and abstract from the possibility of optimal intertemporal consumption smoothing through savings. The time path of the environmental public good is denoted $E_t$. Furthermore, all exogenously provided income $Y_t$ is consumed at each point in time and has to be paid for at given market prices $P_t$, i.e. $C_t = Y_t / P_t$. In the remainder of the paper, we set $P_t = 1$. We

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1The (constant) income elasticity of WTP for the environmental public good is given by $1/\theta$. 

therefore refer to the distribution of consumption $C$ and income $Y$ interchangeably and substitute income $Y$ for the level of consumption of market goods $C$ in the remainder of this paper. Where appropriate, we refer to their distribution over time as ‘intertemporal distribution’, and to the distribution over households as ‘intragenerational distribution’.

As exemplary time paths for income and the environmental public good we study

$$Y_i^t = Y_i^0 (1 + g_Y)^t$$

$$E_t = E_0 (1 + g_E)^t$$

where $Y_i^0$ is household $i$’s levels of income and thus private consumption in period $t = 0$, $E_0$ is the level of the environmental good in period $t = 0$, $g_E \in (-1, 0]$ is the growth rate of the environmental good, and $g_Y \geq 0$ is the growth rate of income. We demonstrate in Appendix A.1 how the time-constant income growth rate, $g_Y$, can be derived as the balanced growth path of a general equilibrium endogenous growth model. As in Baumgärtner et al. (2017) we moreover assume that consumption in $t = 0$ is log-normally distributed over households $i$

$$Y_i^0 \propto \text{LN} (\mu_{Y_0}, \sigma_{Y_0}),$$

where $\mu_{Y_0}$ is the mean level of consumption in society at $t = 0$, and $\sigma_{Y_0}$ is the standard deviation of consumption in $t = 0$. There is empirical evidence that the income distribution can be approximated with a log-normal distribution (Pinkovskiy and Sala-i-Martin 2009), and this has also been assumed to study the related issue of inequality and discounting (Emmerling et al. 2017).

To measure inequality, we focus on the coefficient of variation of consumption $CV_{Y_0} := \frac{\sigma_{Y_0}}{\mu_{Y_0}}$ as a measure of relative income inequality in society. It captures the width of the distribution of income relative to mean income. While there are a number of different notions of income inequality in use, concepts of relative income inequality—often in the form of income shares—features prominently in academic and policy circles, such as in the recent World Inequality Report (Alvaredo et al. 2018). Note that our model set-up
so far makes the assumption that the growth rate of consumption is the same for all households, i.e. $g^i_C = g_Y$. This implies that absolute income inequality—as measured by the standard deviation, for example—will increase over time, while relative income inequality, as measured by the $CV$, will stay constant. In particular, income at each later point in time, $Y_t$, is also log-normally distributed.

In our benchmark model, households have the same pure time preferences and household $i$’s intertemporal utility is given as aggregated discounted instantaneous utility

$$U^i({\{Y^i_t}\}, {\{E_t}\}) = \sum_{t=0}^{\infty} \rho^t \frac{1}{1-\eta} u^i(Y^i_t, E_t)^{1-\eta},$$  \hspace{1cm} (3)$$

where $0 < \rho < 1$ is the pure time discount factor and $\eta$, with $0 \leq \eta < \infty$, is the inverse of the constant intertemporal elasticity of substitution with respect to the within-period aggregate consumption bundle, composed of $E$ and $Y$. Thus, we consider a setting in which each (dynastic) household only cares about the consumption of its own dynasty, has a preference against inequality in comprehensive consumption over time but only a limited altruism towards future selves or descendents (see Asheim and Nesje (2016) for a discussion of intergenerational altruism).

As the measure of economic value for household $i$, we consider the time path $\{x^i_t\}$ of compensating surplus for a change in the time path of environmental goods from $\{E_t\}$ to $\{E'_t\}$.\footnote{In a similar fashion, one can consider the equivalent surplus.} We often just consider the willingness-to-pay (WTP) for a marginal improvement of $\{E_t\}$ instead of compensating surplus, as WTP features more prominently in the (applied) environmental valuation literature. In general, we measure the value of environmental good in units of the market consumption good ($C=Y$):

$$U^i({\{Y^i_t - x^i_t\}, \{E'_t\}}) = U^i({\{Y^i_t\}, \{E_t\}})$$

The time path $x^i_t$ is not a scalar, and for general preferences it is not uniquely defined. We therefore add more structure to our model to be able to capture compensating surplus or WTP as a scalar. One may consider the problem of the household as a hypothetical
choice problem (Neary and Roberts 1980, Hanemann 1991, Flores and Carson 1997), where each household maximizes its intertemporal utility subject to each period’s intratemporal budget constraint, $C_t = Y_t$, as well as the exogenously fixed levels of the environmental good $E_t$ and the market consumption good $C_t$:

$$\max_{\{C_i\},\{E_t\}} U^i(\{C^i_t\}, \{E_t\}) \text{ s.t. } C^i_t = Y^i_t \text{ and } E_t \text{ fixed.}$$ (5)

A household’s income-equivalent valuation of the environmental good is the valuation per unit (Lindahl price), i.e. compensating surplus or WTP, times the level of the environmental good. We study compensating surplus or WTP for two different types of environmental policies, so that $\{E_t\}$ and $\{E_t'\}$ differ in either $E_0$ or $g_E$. When the environmental good differs only in $E_0$, but not in $g_E$, we refer to this as a (marginal) change in the stock of the environmental good. This might be, for instance, an increase in forest cover or a small re-establishment of a species. When the environmental good differs in $g_E$ but not in $E_0$, we refer to this as a (marginal) change in the growth rate of the environmental good. This might be for instance protecting breeding or enhancing nursing ground for pollinators or birds, or slowing down coral bleaching.

To increase the tractability of the model and facilitate closed-form analytic solutions, we assume that the inverse of the constant intertemporal elasticity of substitution with respect to the aggregate consumption bundle, $\eta$, equals the inverse of the elasticity of substitution between market consumption goods and environmental goods, $1/\theta$. This assumption follows theoretic work by Quaas and Bröcker (2016), who build a solvable analytic climate-economy model that extends upon the previous Cobb-Douglas cases in the literature (cf. Golosov et al. 2014). While there is no apparent reason why the assumption $\eta = 1/\theta$ should be fulfilled, there is considerable scope for it to hold if we consider suggested values for $\eta$, which range from 0 to 5 (e.g. Drupp et al. 2018a, Groom and Maddison 2018), and those for $1/\theta$, which might range from 0.14 to 2 (e.g. Drupp 2018, Sterner and Persson 2008).

For these assumptions we obtain the intertemporal utility function for the initial levels and growth rates of the market consumption good, or income respectively, and of
the environmental public good (Appendix A.2):

\[ U^i(\{Y^i_t\}, \{E_t\}) = \frac{\theta}{\theta - 1} \left( \frac{\alpha Y^i_0^{\frac{\theta-1}{\theta}}}{1 - \rho(1 + g_Y)^{\frac{\theta-1}{\theta}}} + \frac{(1 - \alpha) E_0^{\frac{\theta-1}{\theta}}}{1 - \rho(1 + g_E)^{\frac{\theta-1}{\theta}}} \right). \] (6)

This intertemporal utility function (Eq. (6)) only exist for time paths for which the following conditions for the growth rates hold

\[ \rho(1 + g_Y)^{\frac{\theta-1}{\theta}} < 1, \] (7a)

\[ \rho(1 + g_E)^{\frac{\theta-1}{\theta}} < 1. \] (7b)

### 3 Valuation concepts

We now analyze the individual and societal valuation for the environmental good for different payment schemes and objects of valuation. While we sometimes consider the more general measure of compensating surplus, most of our analysis focusses on WTP and societal mean WTP, denoted WTP. We consider WTP for two payment types, pt: (i) a single payment in the initial time period (\(pt = SP\)), and (ii) a constant payment fraction of consumption or income over time (\(pt = CPF\)).\(^3\) Moreover, for each payment scheme we study WTPs for two different environmental policies that induces changes in the stream of the environmental good, eg: marginal changes in the both (a) the level (\(eg = dE\)) and (b) the growth rate (\(eg = dgE\)) of the environmental good. Thus we have four cases, with two payment schemes and two different changes of the environmental good (see Table 1).

\(^3\)While the dynastic household has a preference for consumption smoothing over time, our model abstracts from savings. Therefore, as noted before, a household’s amount of the consumption good is equal to disposable household income (\(C = Y\)).
Table 1: Overview of the four studied WTP cases

<table>
<thead>
<tr>
<th>Payment scheme</th>
<th>Change in natural capital</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>marginal change ( dE ) in</td>
</tr>
<tr>
<td></td>
<td>initial stock ( E_0 )</td>
</tr>
<tr>
<td></td>
<td>marginal change ( dg_E )</td>
</tr>
<tr>
<td>single payment (SP) at t=0</td>
<td>(1) WTP(_{SP,dE})</td>
</tr>
<tr>
<td>constant payment fraction (CPF)</td>
<td>(3) WTP(_{CPF,dE})</td>
</tr>
<tr>
<td></td>
<td>(2) WTP(_{SP,dg_E})</td>
</tr>
<tr>
<td></td>
<td>(4) WTP(_{CPF,dg_E})</td>
</tr>
</tbody>
</table>

3.1 Individual Valuation

We consider two specific cases for compensating surplus or WTP that are prevalent in
the literature: First, a payment to be made in a single period only (hereafter: single
payment or SP), usually in the initial period \( t = 0 \). Second, a payment to be made as
a relative fraction of consumption in each period (hereafter: constant payment fraction
or CPF). Both payment types are used in the applied valuation literature, while CPF
accounts for the bulk of payment vehicles used, often via taxes.\(^4\)

First, if we assume standard time preferences and that the single payment is made
in period \( t = 0 \) only, the compensating surplus \( \{x^i_t\} = \{x^i_{SP}, 0, \ldots\} \) is defined as

\[
\frac{1}{1-\eta} u^i(Y^i_0 - x^i_{SP}, E'_0) + \frac{1}{1-\eta} \sum_{t=1}^{\infty} \rho^t u^i(Y^i_t, E'_t) = \frac{1}{1-\eta} \sum_{t=0}^{\infty} \rho^t u^i(Y^i_t, E'_t)
\]

(8)

Second, another straightforward way to measure the compensating surplus as a scalar
is to consider a constant payment fraction \( x^i \) by which consumption is reduced in each
period: \( \{x^i_t\} = \{(1 - x^i_{CPF}) Y^i_t\} \).

\[
\frac{1}{1-\eta} \sum_{t=0}^{\infty} \rho^t u^i((1 - x^i_{CPF}) Y^i_t, E'_t) = \frac{1}{1-\eta} \sum_{t=0}^{\infty} \rho^t u^i(Y^i_t, E'_t)
\]

(9)

\(^4\)For instance, of the 98 reported WTP values elicited with stated preference methods (‘contingent
valuation’, ‘choice modeling’) in the TEEB-Database (Van der Ploeg and de Groot 2010) 91 WTP-
estimates have been elicited as annual payments (categories ‘annual’ and ‘annual (range)’) while 3
WTP-estimates have been elicited as single payments (category ‘one time payment/ WTP’).
As a first step, we compute compensating surplus for both a single payment as well as for a constant payment fraction. To avoid notational overload, we suppress the index for individual households \(i\) for now.

Regarding a \textit{single payment} in the initial period (Eq. (8)) the compensating surplus, \(x\), is determined by (see Appendix A.3)

\[
x_{\text{SP}} = Y_0 \left( 1 - \left( 1 + Y_0^{1-\theta} \left( \frac{1-\alpha}{\alpha} E_0^{\theta-1} - \frac{1-\alpha}{\alpha} E_0'^{\theta-1} \right) \right)^{\frac{\theta}{\theta-1}} \right).
\]

The compensating surplus does not depend on the growth rate of consumption in this case. This is due to the assumption \(\eta = \frac{1}{\theta}\), which links the preference for intertemporal inequality aversion in consumption and the substitutability between goods at each point in time.\(^5\)

Regarding a \textit{constant payment fraction} (Eq. (9)) the compensating surplus, \(x\), is determined by (see Appendix A.4)

\[
x_{\text{CPF}} = 1 - \left( \frac{1 - \rho (1 + g_E)^{\theta-1}}{\alpha Y_0^{\theta-1}} \left( \frac{(1 - \alpha) E_0'^{\theta-1}}{1 - \rho (1 + g_E')^{\theta-1}} - \frac{(1 - \alpha) E_0'^{\theta-1}}{1 - \rho (1 + g_E')^{\theta-1}} \right) \right)^{\frac{\theta}{\theta-1}}.
\]

We are now equipped to derive the four possible cases of WTP (cf. Table 1), distinguished by the mode of payment (single vs. constant fraction) and whether the marginal change in the environmental goods affects only the initial level or the growth rate.

First, WTP measured as a \textit{single payment} at \(t = 0\) for a marginal change in the level of the environmental good, \(E'_0 = E_0 + dE\), that is leaving the growth path unchanged \(g_E = g'_E\) is given by (see Appendix A.5)

\[
\text{WTP}_{\text{SP},dE} = \frac{1 - \alpha}{\alpha} \frac{Y_0^{1/\theta} E_0^{-1/\theta}}{1 - \rho (1 + g_E)^{\theta-1}} dE.
\]

This is a direct generalization of Baumgärtner et al. (2017) and Ebert (2003).

\(^5\)This may also be different in a setting with endogenous saving decisions and where WTP is large enough in relation to aggregate income. In a case with endogenous savings, one would need to assume that the environmental good only has a marginal contribution to overall welfare (cf. Gollier (2017)).
Second, the WTP for a marginal change in the growth rate of the environmental good, \( g'_E = g_E + dg_E \), that is leaving the level of the environmental good unchanged, \( E_0 = E'_0 \), measured as a single payment is (see Appendix A.6)

\[
WTP_{SP,dgE} = \frac{1 - \alpha}{\alpha} E_0^{\frac{1}{1/\theta}} Y_0^{1/\theta} \frac{\rho(1 + g_E)^{-1/\theta}}{\left(1 - \rho(1 + g_E)^{\frac{1}{1/\theta}}\right)^2} dg_E. \tag{13}
\]

Third, the WTP measured as a constant payment fraction for a marginal change in the initial environmental good, \( E'_0 = E_0 + dE \) and \( g_E = g'_E \), is given by (see Appendix A.7)

\[
WTP_{CPF,dE} = \frac{1 - \alpha}{\alpha} \frac{1 - \rho (1 + g_Y)^{\frac{1}{1/\theta}}}{1 - \rho (1 + g_E)^{\frac{1}{1/\theta}}} Y_0^{1 - \theta} E_0^{1/\theta} dE. \tag{14}
\]

Fourth, assuming \( g'_E = g_E + dg_E \) and \( E_0 = E'_0 \), we derive the WTP, \( WTP_{CPF,dgE} \), as a constant payment fraction for a marginal change of the growth rate of environmental goods (see Appendix A.8)

\[
WTP_{CPF,dgE} = \frac{1 - \alpha}{\alpha} \frac{\rho (1 + g_E)^{-1/\theta} \left(1 - \rho (1 + g_Y)^{\frac{1}{1/\theta}}\right)}{\left(1 - \rho (1 + g_E)^{\frac{1}{1/\theta}}\right)^2} Y_0^{1 - \theta} E_0^{\frac{1}{1/\theta}} dE. \tag{15}
\]

### 3.2 Societal Valuation

We now turn to aggregating individual WTPs within a society. Mean WTP in terms of a single payment at \( t = 0 \) for the environmental good at level \( E \), \( \overline{WTP}_{SP,dE} \), is given by

\[
\overline{WTP}_{SP,dE}(\mu_{Y_0}, \sigma_{Y_0}) = \int_0^\infty f_{\ln}(Y_0; \mu_{Y_0}, \sigma_{Y_0}) WTP_{SP,dE}(Y_0) dY_0, \tag{16}
\]

where \( f_{\ln}(Y_0; \mu_{Y_0}, \sigma_{Y_0}) \) is the density function of the log-normal distribution of initial income \( Y_0 \) with mean \( \mu_{Y_0} \) and standard deviation \( \sigma_{Y_0} \). Compensating surplus for a single payment in \( t = 0 \) is given by (12). Then mean WTP in terms of a single payment at \( t = 0 \) (Eq. (16)) can be reformulated as (see Appendix A.9)
\[ \text{WTP}_{\text{SP},dE}(\mu Y_0, CV Y_0) = \kappa \mu Y_0^{1/\theta} \left(1 + CV Y_0^2\right)^{1-\theta/2} \]

with \( \kappa = \frac{1 - \alpha}{\alpha} \frac{E_0^{-1/\theta} dE}{1 - \rho (1 + gE)^{\theta/\sigma}} \),

where \( CV Y_0 \) is the initial relative income inequality.

The corresponding mean WTP as single payment for a marginal change in the growth rate is given by (see Appendix A.10)

\[ \text{WTP}_{\text{SP},dgE}(\mu Y_0, CV Y_0) = \kappa' \mu Y_0^{1/\theta} \left(1 + CV Y_0^2\right)^{1-\theta/2} \]

with \( \kappa' = \frac{1 - \alpha}{\alpha} \frac{E_0^{\theta/\sigma-1} \rho(1 + gE)^{-1/\theta}}{1 - \rho(1 + gE)^{\theta/\sigma}} dgE. \)

Thus, the value of the level of the environmental good elicited as a single payment in \( t = 0 \) (Eq. (17) and Eq. (18)) does not depend on income growth, \( gY \). The mean WTP function obtained (Eq. (17)) is structurally identical to the one in the static setting obtained by Baumgartner et al. (2017), with differences in \( \kappa \).

Next, we turn to societal WTP elicited as a constant payment fraction. Note that we now have to multiply the CPF with the respective level of income in each period, with \( Y_t = (1 + gY)^t Y_0 \), to obtain the overall mean WTP. While individual time preferences affect the individual CPF, the planner applies her own discount rate when aggregating yearly mean WTPs over time. The planner’s discount rate may be given by the (risk-free) market discount rate, where \( \delta_\tau \) is the interest factor at time \( \tau \).

The (undiscounted) mean WTP at time \( t \) for a marginal change in \( E \) is

\[ \text{WTP}_{\text{CPF},dE,t}(\mu Y_0, \sigma Y_0) = \int_0^\infty f_{\ln}(Y_0; \mu Y_0, \sigma Y_0) \text{WTP}_{\text{CPF},dE}(Y_0)(1 + gY)^t Y_0 dY_0. \]

Mean WTP at time \( t \) as constant payment fraction for a marginal change in \( E \), \( \text{WTP}_{\text{CPF},dE,t} \)

---

6We show in Appendix A.1 how the time-constant interest factor, \( \delta_t = \delta \), can be derived from an one-sector endogenous growth model.
(Eq. (14)), can be written as a function of the moments of the income distribution as follows (see Appendix A.11)

$$
WTP_{CPF,dE:t}(\mu_{Y_0}, CV_{Y_0}) = \kappa'' \mu_{Y_0}^{1/\theta} \left(1 + CV_{Y_0}^2\right)^{\frac{1-\theta}{2\theta^2}} (1 + gY)^t \left(1 - \rho(1 + gY)^{\frac{\theta-1}{\theta}}\right) E_0^{-1/\theta} dE.
$$

(20)

with

$$
\kappa'' = \frac{1 - \alpha}{\alpha} \frac{1 - \rho(1 + gY)^{\frac{\theta-1}{\theta}}}{1 - \rho(1 + gE)^{\frac{\theta-1}{\theta}}} E_0^{-1/\theta} dE.
$$

The associated present value of the WTP as a constant payment fraction for a marginal change in the environmental good - discounted at market interest rates - is given by:

$$
WTP_{CPF,dE}(\mu_{Y_0}, CV_{Y_0}) = \sum_{t=0}^{\infty} \left(\prod_{\tau=0}^{t} \delta_{\tau}\right) WTP_{CPF,dE:t}(\mu_{Y_0}, CV_{Y_0}).
$$

(21)

This can be reformulated as (see Appendix A.11):

$$
WTP_{CPF,dE}(\mu_{Y_0}, CV_{Y_0}) = \kappa'' \mu_{Y_0}^{1/\theta} \left(1 + CV_{Y_0}^2\right)^{\frac{1-\theta}{2\theta^2}} (1 + gY)^t \left(1 - \rho(1 + gY)^{\frac{\theta-1}{\theta}}\right) E_0^{-1/\theta} dE \left[\sum_{t=0}^{\infty} \left(\prod_{\tau=0}^{t} \delta_{\tau}\right) (1 + gY)^t\right].
$$

(22)

Analogously, the mean WTP at time $t$ as constant payment fraction for a marginal change in the growth rate, $WTP_{CPF,dgE:t}$, is given by (see Appendix A.12):

$$
WTP_{CPF,dgE:t}(\mu_{Y_0}, CV_{Y_0}) = \kappa''' \mu_{Y_0}^{1/\theta} \left(1 + CV_{Y_0}^2\right)^{\frac{1-\theta}{2\theta^2}} (1 + gE)^t \left(1 - \rho(1 + gE)^{\frac{\theta-1}{\theta}}\right) dE (1 + gY)^t E_0^{-1/\theta}.
$$

(23)

with

$$
\kappa''' = \frac{1 - \alpha}{\alpha} \frac{\rho(1 + gE)^{-1/\theta} \left(1 - \rho(1 + gY)^{\frac{\theta-1}{\theta}}\right)}{(1 - \rho(1 + gE)^{\frac{\theta-1}{\theta}})^2} dE (1 + gY)^t E_0^{-1/\theta},
$$

and the associated present value is given by (see Appendix A.12):

$$
WTP_{CPF,dgE}(\mu_{Y_0}, CV_{Y_0}) = \sum_{t=0}^{\infty} \left(\prod_{\tau=0}^{t} \delta_{\tau}\right) WTP_{CPF,dgE:t}(\mu_{Y_0}, CV_{Y_0})
$$

$$
= \kappa''''' \mu_{Y_0}^{1/\theta} \left(1 + CV_{Y_0}^2\right)^{\frac{1-\theta}{2\theta^2}} (1 + gE)^t \left(1 - \rho(1 + gE)^{\frac{\theta-1}{\theta}}\right) dE (1 + gY)^t E_0^{-1/\theta} \sum_{t=0}^{\infty} \left(\prod_{\tau=0}^{t} \delta_{\tau}\right) (1 + gY)^t.
$$

(24)
4 Results

In this section, we study how a change in (i) mean income, $\mu_{Y_0}$, or (ii) intragenerational income inequality, $CV_{Y_0}$, affects societal WTP for an increase in the stock or the growth rate of the environmental public good (Eq. (17), (18), (22), (24)). Moreover, we study how a change in (iii) the growth rate of income, $g_Y$, determining the intergenerational distribution of income, or (iv) the growth rate of the environmental good, $g_E$, affects societal WTP measured as a constant payment fraction for an increase in the stock of the environmental public good (Eq. (22)). Finally, we (v) derive adjustment factors for applications such as benefit transfer, environmental cost-benefit analysis or natural capital accounting. We address these five tasks in turn.

First, how does society’s current mean income affect the societal intertemporal mean WTP measured as a single payment or a constant payment fraction for a change in the level or the growth rate of the environmental public good?

Proposition 1
Mean WTP elicited as a single payment or a constant payment fraction for an increase in the level or the growth rate of the environmental public good—$WTP_{SP,dE}$ (Eq. (17)), $WTP_{SP,dgE}$ (Eq. (18)), $WTP_{CPF,dE}$ (Eq. (22)), and $WTP_{CPF,dgE}$ (Eq. (24))—increases with mean income, $\mu_{Y_0}$:

$$\frac{\partial WTP_{pt,eg}(\mu_{Y_0}, CV_{Y_0}, g_Y, g_E)}{\partial \mu_{Y_0}} > 0.$$  (25)

Proof. See Appendix A.13. \qed

Proposition 1 states that the effect of societies (initial) mean income on societal WTP is unambiguous: Mean WTP for the stock or the growth rate of the environmental good increase with mean income. Proposition 1 generalizes the result from the static setting obtained in Baumgärtner et al. (2017) to a dynamic setting and to different objects of valuation.
Second, how does society’s current relative income inequality affect the societal intertemporal mean WTP measured as a single payment or a constant payment fraction for a change in the level or the growth rate of the environmental public good?

Proposition 2

Mean WTP elicited as a single payment or a constant payment fraction for an increase in the level or the growth rate of the environmental public good—\( \text{WTP}_{\text{sp},dE} \) (Eq. (17)), \( \text{WTP}_{\text{sp},dgE} \) (Eq. (18)), \( \text{WTP}_{\text{cpf},dE} \) (Eq. (22)), \( \text{WTP}_{\text{cpf},dgE} \) (Eq. (24))—decreases (increases) with relative intragenerational income inequality, \( \text{CV}_{Y_0} \), if and only if the environmental public good and the private consumption good are substitutes (complements):

\[
\frac{\partial \text{WTP}_{\text{pt, eg}}(\mu_{Y_0}, CV_{Y_0}, g_Y, g_E)}{\partial CV_{Y_0}} \leq 0 \quad \text{if and only if} \quad \theta \geq 1. \tag{26}
\]

Proof. See Appendix A.14.

Proposition 2 states that (initial) relative intragenerational income inequality affects mean WTP for natural capital and that the sign of the effect depends on whether the environmental public goods derived from natural capital are a substitute or a complement to market-traded manufactured consumption goods. If they are substitutes, mean WTP for natural capital decreases with income inequality. If they are complements, mean WTP for natural capital increases with income inequality. Proposition 2 generalizes the central finding in Baumgärtner et al. (2017) to an intertemporal setting: The degree of substitutability is the key determinant of how intra-temporal income inequality affects societal WTP. Having established these two findings for the intra-temporal distribution, we now turn to scrutinizing the intertemporal distribution and specifically examine how growth rates affect WTP.

Third, how does the intergenerational distribution - given by the growth rate of income - affect societal intertemporal mean WTP measured as a constant payment fraction for a change in the level of the environmental good?
Proposition 3
Mean WTP elicited as a constant payment fraction for an increase in the level of the environmental public good—$\text{WTP}_{CPF,dE}$ (Eq. (22))—for a time-constant market interest factor, $\delta < \frac{1}{1+g_Y}$, increases with the growth rate of income, $g_Y$, if the environmental public good and the private consumption good are complements or Cobb-Douglas:

$$\frac{\partial \text{WTP}_{CPF,dE} (\mu_{Y_0}, CV_{Y_0}, g_Y, g_E)}{\partial g_Y} > 0 \text{ if } \theta \leq 1.$$ (27)

Proof. See Appendix A.15.

How the intergenerational distribution of income and wealth affects societal WTP depends on the level of the growth rate of income, the relative sizes of the pure time discount and market interest factors as well as the degree of substitutability. For the special case of Cobb-Douglas substitutability as well as for the case of complements, we find that an increase in intergenerational inequality in terms of consumption goods increases societal WTP for the public environmental good. However, if the environmental good is a substitute to manufactured goods there are cases where an increase in intergenerational inequality in terms of consumption goods leads to a decrease of societal WTP, depending on the relative magnitudes of the elasticity of substitution, the growth rate of income as well as pure time and market interest rate factors. We illustrate the range of conditions for which WTP for a substitutable environmental good may fall with the growth rate of income in Figure 7 in Appendix A.16.

Societal WTP elicited as a single payment - $\text{WTP}_{SP,dE}$ (Eq. (17)) and $\text{WTP}_{SP,dgE}$ (Eq. (18)) - does not depend on the growth rate of income, $g_Y$. This implies that WTP measured as a single payment is generally not affected by a change in the $g_Y$.

Forth, how does the intergenerational distribution - given by the growth rates of the environmental good - affect societal intertemporal mean WTP measured as a single payment or constant payment fraction for a change in the level of the environmental good?
Proposition 4

Mean WTP elicited as a single payment or a constant payment fraction for an increase in the level of the environmental public good—$\overline{WTP}_{SP,dE}$ (Eq. (17)), $\overline{WTP}_{CPF,dE}$ (Eq. (22))—increases (decreases) with the growth rate of the environmental good, $g_E$, if and only if the environmental public good and the private consumption good are substitutes (complements):

$$\frac{\partial \overline{WTP}_{CPF,dE}(\mu_{Y_0}, CV_{Y_0}, g_Y, g_E)}{\partial g_E} \gtrless 0 \text{ if and only if } \theta \gtrless 1. \quad (28)$$

Proof. See Appendix A.17.

The intergenerational distribution of natural capital, captured by the growth rate of environmental goods, does not affect societal WTP only for the special case of Cobb-Douglas substitutability. For substitutes, an increase in intergenerational inequality in terms of environmental goods increases societal WTP, while it is the reverse case when the environmental good is a complement to manufactured goods.

The growth rate of the environmental good determines how scarce environmental goods become over time. The resulting change in value – as captured by the societal WTP – thereby strongly depends on how well these environmental goods can be substituted with manufactured consumption goods .... For environmental goods being substitutes to consumption goods mean WTP increases with their growth rate leading to a world where environmental goods are relatively more plentiful, but decreases when environmental goods are becoming more scarce over time. To the contrary, if it is more difficult to substitute environmental goods

Fifth, how should one adjust societal intertemporal mean WTP measured as a single payment or a constant payment fraction for a change in the level or the growth rate of the environmental public good for differences in the distribution of income when conducting value transfer from a study to a policy site?
We now derive adjustment factors for site specific differences in the distribution of income, growth rates and interest rates. Benefit transfer has become a primary method of environmental valuation (Richardson et al. 2015) and “the bedrock of practical policy analysis” (Pearce et al. 2006: 266) to inform government decision making. As most of the benefit transfer literature and practice employs predominantly empirical approaches, there have been calls to base benefit transfers approaches more firmly in economic theory (Bateman et al. 2011). Several government guidelines for economic appraisal already propose to use an explicit transfer factor to account for differences in mean income between the study context of the primary valuation (‘study site’) and the decision making contest (‘policy site’), e.g. in Germany (UBA 2012) and the UK (Defra 2007). This was complemented and taken further by Baumgärtner et al. (2017), who provided additional theory-driven adjustment factors, in particular for income inequality. Empirical evidence from a multi-country valuation study shows that employing this theory-driven adjustment factor for income inequality increases the accuracy of benefit transfers (Meya et al. 2017).

With the model setting presented here we can show that these transfer factors for differences in the income distribution also hold more generally in a dynamic setting and we derive additional transfer factors for growth rates and market interest rates. Thereby, we specify the benefit transfer function (e.g. Loomis 1992) to explicitly account for the time dimension. These generalizations and extensions make the benefit function approach more suitable for natural capital accounting. Mean WTP for a policy site, \( \bar{WTP}_{\text{policy}} \), can then be estimated as the product of a simple transfer function \( T \) with the mean WTP elicited at a study site, \( \bar{WTP}_{\text{study}} \).

**Proposition 5**

Assume that households’ preferences \((\theta, \alpha, \rho)\) are identical in the study and the policy sites. Mean WTP as a single payment for a marginal change in the level of the environmental public good in a policy site—\( \bar{WTP}_{\text{policy}SP, \Delta E} \), \( \bar{WTP}_{\text{policy}SP, \Delta gE} \), \( \bar{WTP}_{\text{policy}CPF, \Delta E} \), and \( \bar{WTP}_{\text{policy}CPF, \Delta gE} \)—is given by
with the following transfer function:

\[
\mathcal{T}_{SP,dE}(\ldots) = \mathcal{T}_{E}(E_0^{\text{policy}}, E_0^{\text{study}}, \theta) \cdot \mathcal{T}_{dE}(dE^{\text{policy}}, dE^{\text{study}}) \cdot \mathcal{T}_{gE}(g_E^{\text{policy}}, g_E^{\text{study}}, \theta, \rho) \cdot \mathcal{T}_{\mu}(\mu_0^{\text{policy}}, \mu_0^{\text{study}}, \theta) \cdot \mathcal{T}_{CV}(CV_Y^{\text{policy}}, CV_Y^{\text{study}}, \theta),
\]

(30)

The mean WTPs, $\overline{\text{WTP}}_{\text{pt,eg}}$ for the other three cases of payment types and environmental good changes yield the following transfer functions for transferring $\overline{\text{WTP}}_{\text{pt,eg}}$ into $\overline{\text{WTP}}_{\text{policy}}$:

\[
\mathcal{T}_{SP,dgE}(\ldots) = \mathcal{T}_{E}(E_0^{\text{policy}}, E_0^{\text{study}}, \theta) \cdot \mathcal{T}_{dgE}(dg_E^{\text{policy}}, dg_E^{\text{study}}) \cdot \mathcal{T}_{gE}(g_E^{\text{policy}}, g_E^{\text{study}}, \theta, \rho) \cdot \mathcal{T}_{\mu}(\mu_0^{\text{policy}}, \mu_0^{\text{study}}, \theta) \cdot \mathcal{T}_{CV}(CV_Y^{\text{policy}}, CV_Y^{\text{study}}, \theta),
\]

(31)

\[
\mathcal{T}_{CPF,dE}(\ldots) = \mathcal{T}_{E}(E_0^{\text{policy}}, E_0^{\text{study}}, \theta) \cdot \mathcal{T}_{dE}(dE^{\text{policy}}, dE^{\text{study}}) \cdot \mathcal{T}_{gE}(g_E^{\text{policy}}, g_E^{\text{study}}, \theta, \rho) \cdot \mathcal{T}_{\mu}(\mu_0^{\text{policy}}, \mu_0^{\text{study}}, \theta) \cdot \mathcal{T}_{CV}(CV_Y^{\text{policy}}, CV_Y^{\text{study}}, \theta) \cdot \mathcal{T}_{gY,\delta_j}(g_Y^{\text{policy}}, g_Y^{\text{study}}, \delta_Y^{\text{policy}}, \delta_Y^{\text{study}}, \theta, \rho),
\]

(32)

\[
\mathcal{T}_{CPF,dgE}(\ldots) = \mathcal{T}_{E}(E_0^{\text{policy}}, E_0^{\text{study}}, \theta) \cdot \mathcal{T}_{dgE}(dg_E^{\text{policy}}, dg_E^{\text{study}}) \cdot \mathcal{T}_{gE}(g_E^{\text{policy}}, g_E^{\text{study}}, \theta, \rho) \cdot \mathcal{T}_{\mu}(\mu_0^{\text{policy}}, \mu_0^{\text{study}}, \theta) \cdot \mathcal{T}_{CV}(CV_Y^{\text{policy}}, CV_Y^{\text{study}}, \theta) \cdot \mathcal{T}_{gY,\delta_j}(g_Y^{\text{policy}}, g_Y^{\text{study}}, \delta_Y^{\text{policy}}, \delta_Y^{\text{study}}, \theta, \rho),
\]

(33)
The corresponding disentangled transfer factors are given by:

\[
T_E^{(dE)}(E_0^{policy}, E_0^{study}; \theta) = \left( \frac{E_0^{policy}}{E_0^{study}} \right)^{-1/\theta},
\]

(34)

\[
T_E^{(dgE)}(E_0^{policy}, E_0^{study}; \theta) = \left( \frac{E_0^{policy}}{E_0^{study}} \right)^{\theta-1},
\]

(35)

\[
T_{dE}(dE^{policy}, dE^{study}) = \frac{dE^{policy}}{dE^{study}},
\]

(36)

\[
T_g^{(dE)}(g_E^{policy}, g_E^{study}; \theta, \rho) = \frac{1 - \rho(1 + g_E^{study})^{\theta-1} + \rho(1 + g_E^{policy})^{\theta-1}}{1 - \rho(1 + g_E^{policy})^{\theta-1}},
\]

(37)

\[
T_{gE}(dE^{policy}, dE^{study}) = \frac{dg_E^{policy}}{dg_E^{study}},
\]

(38)

\[
T_{CV}(CV_Y^{policy}, CV_Y^{study}; \theta) = \left( \frac{1 + CV_Y^{policy}}{1 + CV_Y^{study}} \right)^{1 - \frac{1-\theta}{2\theta^2}},
\]

(39)

\[
T_{gY, \delta}^{(gY, \delta)}(g_Y^{policy}, \delta_Y^{policy}, g_Y^{study}, \delta_Y^{study}; \theta, \rho) \nonumber
\]

\[
= \frac{1 - \rho(1 + g_Y^{policy})^{\theta-1} + \rho(1 + g_Y^{study})^{\theta-1}}{1 - \rho(1 + g_Y^{study})^{\theta-1} + \rho(1 + g_Y^{policy})^{\theta-1}} \cdot \sum_{t=0}^{\infty} \left( \prod_{\tau=0}^{t} \delta_Y^{policy} \right) (1 + g_Y^{policy})^t \sum_{t=0}^{\infty} \left( \prod_{\tau=0}^{t} \delta_Y^{study} \right) (1 + g_Y^{study})^t.
\]

(40)

Proof. See Appendix A.18.

Proposition 5 develops a set of four specific transfer functions for different payment vehicles and objects of valuation. It shows that adjustment for differences in the income distribution can be done in the same way for all four cases by exploiting information on the intragenerational income distribution. Thereby, \(T_{CV}(CV_Y^{policy}, CV_Y^{study}; \theta)\) and \(T_{\mu}(\mu_Y^{policy}, \mu_Y^{study}; \theta)\) make the results in Baumgärtner et al. (2017) applicable for the specific intertemporal setting considered here. Moreover, Proposition 5 shows that
one has to apply specific transfer factors for differences in the level of the environmental public good or the growth rates—depending on the component of natural capital one seeks to value. Finally, for WTP elicited as a constant payment fraction—which is the more common approach in primary valuation—our dynamic model shows how to adjust mean WTP for differences in income growth and interest rates by using $T_{g_Y, \delta_r}(g_Y^{\text{policy}}, \delta_r^{\text{policy}}, g_Y^{\text{study}}, \delta_r^{\text{study}}, \theta; \rho)$.

Furthermore, our dynamic model provides guidance how to adjust mean WTP for differences in the growth rate of the environmental good by employing $T_{dE}(g_E^{\text{policy}}, g_E^{\text{study}}; \theta, \rho)$ or $T_{dgE}(g_E^{\text{policy}}, g_E^{\text{study}}; \theta, \rho)$ depending on whether a change in the level or the growth rate of the environmental good is valued.

5 Application: Global biodiversity conservation

5.1 Data

Here, we introduce our case study on WTP for global ecosystem services and biodiversity. An overview of the inputs to our empirical application is given in Table 2.

For the initial global income distribution we draw on Pinkovskiy and Sala-i-Martin (2009), who estimate the global per-capita income distribution for 2006 finding a mean of $\mu_{Y_0} = 9,550$ [2006-PPP-USD] and a standard deviation of $\sigma_{Y_0} = 15,400$ [2006-PPP-USD] (Pinkovskiy, personal communication). This corresponds to a coefficient of variation of $CV_{Y_0} = 1.61$. For the forecasted growth rate of income, $g_Y$, we draw on an expert survey by Drupp et al. (2018a). Over two hundred experts on long-term societal decision-making were asked to provide their best guess of the global average, long-term (> 100 years) annual growth rate of real per-capita consumption. They find a mean consumption growth rate of $g_Y^{\text{mean}} = 1.7$ percent. The lower bound (abbreviated as ‘lb’) is -2 percent and the upper bound (‘ub’) is $g_Y^{\text{ub}} = 5$ percent. As only three experts stated a negative growth rates and, in order to stay consistent with our model assumptions, we take $g_Y^{\text{lb}} = 0.1$ percent as lower bound value.
Table 2: Variable and parameter values used in the application

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value(s)</th>
<th>Description</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>CV_Y0</td>
<td>1.61</td>
<td>Coefficient of variation of global per-capita income</td>
<td>Pinkovskiy and Sala-i-Martin (2009)</td>
</tr>
<tr>
<td>µ_Y0</td>
<td>9,550</td>
<td>Mean global per-capita income in 2006-PPP-USD</td>
<td>Pinkovskiy and Sala-i-Martin (2009)</td>
</tr>
<tr>
<td>g_Y</td>
<td>0.017</td>
<td>Annual real per-capita (consumption) growth rate</td>
<td>Drupp et al. (2018a)</td>
</tr>
<tr>
<td>E_0</td>
<td>1</td>
<td>Normalized to one</td>
<td>–</td>
</tr>
<tr>
<td>g_E</td>
<td>[−0.0052; −0.0008]</td>
<td>Growth rate of global non-use environmental goods</td>
<td>Baumgärtner et al. (2015)</td>
</tr>
<tr>
<td>α</td>
<td>0.85 [0.7; 1]</td>
<td>Utility share parameter for consumption goods</td>
<td>Kopp et al. (2012)</td>
</tr>
<tr>
<td>θ</td>
<td>2.31 [0.86; 7.14]</td>
<td>Elasticity of substitution</td>
<td>Drupp (2018)</td>
</tr>
<tr>
<td>ρ</td>
<td>0.989 [0.926; 1]</td>
<td>Pure time discounting factor</td>
<td>Drupp et al. (2018a)</td>
</tr>
<tr>
<td>δ</td>
<td>0.977 [0.94; 1]</td>
<td>Risk-free market interest factor</td>
<td>Drupp et al. (2018a)</td>
</tr>
</tbody>
</table>

Note: Numbers in brackets correspond to lower and upper bound values. We approximate the upper bound discount factor with 0.999 instead of 1.

We normalize the initial level of the global environmental good to E_0 = 1. For the growth rate of the environmental good, g_E, we focus on non-use or non-consumptive ecosystem services, as this provides the best fit for our simple mapping from natural capital to ecosystem service provisioning. We take the global mean annual growth rate of cultural ecosystem services estimated by Baumgärtner et al. (2015), based on the best available time-series data for different ecosystem services and countries. These include as ecosystem service measures landscape connectedness, forest area, living planet index, red-list-index and national biodiversity indicators. Baumgärtner et al. (2015) estimate the global average growth rate as \( g_E^{\text{mean}} = -0.52 \) with a lower bound of \( g_E^{\text{lb}} = -1.28 \) percent and an upper bound of \( g_E^{\text{ub}} = -0.08 \) percent.

We take the preference parameters from the literature, in particular from meta-studies and international expert surveys. For the utility share parameter of environmental goods, α, we draw on the parameter range considered by Kopp et al. (2012), ranging from 0 to 0.3, with a mean of 0.15 for environmental goods and thus from 0.7
to 1 for consumption goods, with a mean of 0.85.\textsuperscript{7} For the elasticity of substitution between the environmental and the market-traded consumption good, we use data from a meta-study by Drupp (2018), who gathers indirect evidence from 18 environmental valuation studies. This yields a mean elasticity of substitution of $\theta^{\text{mean}} = 2.31$, implying that environmental goods are considered as substitutes to market-traded goods on average, with a lower and upper bound of $\theta^{\text{lb}} = 0.86$ and $\theta^{\text{ub}} = 7.14$ respectively.\textsuperscript{8} For the pure time discount factor, the elasticity of marginal utility of consumption and the market interest factor, we again draw on survey data from Drupp et al. (2018a), who elicited expert recommendations and long-run forecasts. They obtain a mean rate of pure time preference of 1.1 percent, with a lower and upper bound of 0 and 8 percent. This translates into an initial mean pure time discount factor of $\rho^{\text{mean}} = 0.989$, with a lower and upper bound of 0.926 and 1.\textsuperscript{9} The mean forecasted real risk-free market interest rate is 2.38 percent, with a lower and upper bound of 0 and 6 percent. This translates into an initial mean market interest factor $\delta^{\text{mean}} = 0.977$, with a lower and upper bound of $\delta^{\text{lb}} = 0.943$ and $\delta^{\text{ub}} = 1$.

We quantify our main results for a time horizon of hundred years, $T = 100$, and a hypothetical one percent increase in the level or growth rate of the environmental good. The corresponding changes $dE = 0.01$ in the level of the environmental good and $dg_E = 0.000052$ in the growth rate of the environmental good that we seek to value are pure scaling factors in the mean WTP-functions (Eq. (12) to (15)).

Before quantifying our main results, by specifying the mean WTP function and the transfer factors, we need to make sure, that the conditions on the growth paths hold (Eq. (7a) and Eq. (7b)). The set of growth rates that meet the existence condition for the closed-form intertemporal utility function for a given elasticity of substitution, $\theta$, and discount factor, $\rho$, is given as\textsuperscript{10}

\footnotesize
\begin{itemize}
  \item This encompasses parameter values chosen by Sterner and Persson (2008), who assume $1 - \alpha = 0.1$, and Gollier (2010), who assumes $1 - \alpha = 0.29$.
  \item Note that values implying stronger complementarity have been used in the applied theory and modelling literature. For example, Sterner and Persson (2008) used a central value of 0.5.
  \item We use a value of 0.999 instead of 1 to ensure that our intertemporal welfare function is bounded.
  \item As the condition is identical for $g_E$ and $g_Y$, we suppress the subscript on the growth rate in the
\end{itemize}

\normalsize
Figure 1: Minimal value for $g_E$ in case of complements, $\theta < 1$, (left side) and maximal value for $g_Y$ in case of substitutes, $\theta > 1$, (right side) to ensure the existence of a closed-form intertemporal utility function (Eq. (3)) for different values of the discount factor, $\rho$, and the elasticity of substitution, $\theta$. The shaded area depicts the set of growth rates $g_E$ (left side) or $g_Y$ (right side) that meets the growth path condition (Eq. (7b) or (7a)) for the transfer factor for the mean pure time discount factor, $\rho = 0.989$.

$$\rho(1 + g)^{\frac{\theta - 1}{\theta}} < 1 \iff \begin{cases} g < \rho^{1-\theta} - 1 =: g_{\text{max}} & \text{for } \theta > 1 \\ g > \rho^{1-\theta} - 1 =: g_{\text{min}} & \text{for } \theta < 1 \end{cases}.$$  \hspace{1cm} (43)

Thus, the growth path condition for substitutes implies a supremum defined by $g_{\text{max}}$, which is always positive and thus bites only for the income growth rate $g_Y$, but not for the growth rate of the environmental good which is by definition always negative, $g_E < 0$. In contrast, the growth rate condition for complements implies a infimum for the growth rates $g_{\text{min}}$, below which the closed-form intertemporal utility function does not exist. As $g_{\text{min}}$ is always negative, this condition is generally fulfilled for the income growth rate, $g_Y > 0$, but applies for the growth rate of the environmental good, $g_E < 0$. Following formula and only write $g$. 

26
Figure 1 displays this frontier for the growth rates of income and the environmental good for a range of empirical elasticities of substitution, $\theta$, and pure time discount factors, $\rho$, depicted in Table 2. Depending on $\rho = 0.989 \; [0.926; 0.999]$ in case of substitutes the supremum for the income growth rate with $\theta_{\text{mean}} = 2.31$ is $g_{\theta=2.31}^{\text{max}} = 0.0197 \; [0.1452; 0.0018]$, and with $\theta^\text{lb} = 7.14$ is $g_{\theta=7.14}^{\text{max}} = 0.0129 \; [0.0935; 0.0012]$. For complements with $\theta^\text{ib} = 0.86$ the infimum for the growth rate of the environmental good is $g_{\theta=0.86}^{\text{min}} = -0.0657 \; [-0.3764; -0.0061]$. We observe that the closer the discount factor $\rho$ is to one, i.e. the closer the discount rate is to zero, the smaller is the set of $g_E$ in case of complements and of $g_Y$ in case of substitutes that fulfils the growth path condition. Moreover, the higher the degree of substitutability, $\theta \to \infty$, the smaller is the set of $g_Y$ that still meets the condition and the stronger the complementarity, $\theta \to 0$, the larger is the set of $g_E$ that meets the condition.

When we compare these minima and maxima for $g_E$ and $g_Y$ for which a closed-form intertemporal utility function exists with empirical data on growth rates, we see that these conditions appear generally uncritical for the rate of loss of ecosystem services $g_E$ but not for income growth rates $g_Y$. For the mean estimate on the discount factor, $\rho_{\text{mean}} = 0.989$, and complements with the strongest complementarity, $\theta^\text{ib} = 0.86$, found in valuation studies as reviewed by Drupp (2018) the minimal growth rate $g_{\theta=0.86}^{\text{min}} = -0.0657$ is well below the lower bound rate of global loss of ecosystem services $g_{E}^{\text{lb}} = -0.0128$ estimated by Baumgärtner et al. (2015). Also for the mean pure time discount factor and the best guess estimate for the degree of substitutability, $\theta_{\text{mean}} = 2.31$, the maximal income growth rate is with $g_{\theta=2.31}^{\text{max}} = 0.0197$ higher than the mean of the long term growth rate expected by international experts of $g_{Y}^{\text{mean}} = 0.017$. Thus, for the main specification of our model the growth path condition is fulfilled. However, the upper bound of the expected annual global income growth rate of $g_{Y}^{\text{ub}} = 0.05$ does not meet the growth path condition for the mean substitutability parameter $g_{\theta=2.31}^{\text{max}}$. Moreover, for the upper bound of the substitutability parameter, $\theta = 7.14$, the mean growth rate, $g_{Y}^{\text{mean}}$, is already higher than the maximal allowed value $g_{\theta=7.14}^{\text{max}}$ and thus growth path condition is not fulfilled.
5.2 Quantification of main results

We estimate how the intra- and intergenerational distribution affects mean WTP for global ecosystem services and biodiversity. Moreover, we compute transfer factors that allow controlling for the intertemporal aspects of natural capital valuation. We focus on the case of mean WTP measured as a constant payment fraction for a marginal change in the environmental good, $WTP_{CPF,dE}$, throughout this subsection, as the bulk of empirical valuation studies falls within this category.

Figure 2 depicts how mean income (left side) and income inequality (right side) affect global mean WTP measured as a constant payment fraction for an increase in global ecosystem services. First of all, it is apparent that the degree of substitutability, $\theta$, is crucial in determining mean WTP, $\bar{WTP}_{CPF,dE}$. For initial global mean income and the mean substitutability estimate $\theta^{\text{mean}} = 2.31$, we obtain a mean WTP of 0.632 [2006-PPP-USD]. However, if ecosystem services were a complement to manufactured consumption goods, $\theta^{lb} = 0.86$, mean WTP would be magnitudes higher and amount to $8.396 \times 10^3$ [2006-PPP-USD]. These estimates highlight that the societal value of global ecosystem services strongly depends on their substitutability: The harder it is to substitute ecosystem services with human-made goods, the higher is their societal value.

Mean WTP for global biodiversity conservation is increasing with mean income (see Proposition 1). For substitutes mean WTP is a strictly increasing concave function of mean income (Figure 2 top left subplot), while it is a convex function for complements (bottom left subplot). For a hypothetical doubling of global per-capita income, mean WTP would be 34.99% higher for the mean substitutability estimate. In this case WTP-estimates would need to be adjusted with a factor of $T_\mu(2\mu_{Y_0}^{GLO}, \mu_{Y_0}^{GLO}, \theta^{\text{mean}}) = 1.35$. For the lower bound range of complements, $\theta^{lb} = 0.86$, it would be even 123.89% higher as initially, corresponding to adjustment factor of $T_\mu(2\mu_{Y_0}^{GLO}, \mu_{Y_0}^{GLO}, \theta^{lb}) = 2.24$.

The subplots on the right of Figure 2 illustrate how mean WTP for global biodiversity conservation changes for a change in relative intragenerational income inequality as measured by the coefficient of variation of per-capita income, $CV_{Y_0}$. While mean WTP decreases with income inequality for the case of substitutes and thus for the mean...
Figure 2: Effect of mean income, $\mu_Y$, (left side) or relative intragenerational income inequality, $CV_Y$, (right side) on the present value mean WTP for a one percent increase in global non-use environmental goods measured as a constant payment fraction, $WTP_{CPF,dE}$, for different degrees of substitutability between the consumption and the environmental good, $\theta$.

empirical case (top right subplot), it increases for complements (bottom right subplot) (see Proposition 2). A hypothetical reduction of the current level of global income inequality, $CV_{GLO} = 1.61$, to zero would increase mean WTP by 17.00% corresponding to a transfer factor of $T_{CV}(0, CV_{GLO}^Y; \theta^{mean}) = 1.17$ given the mean empirical estimate for the elasticity of substitution, $\theta^{mean} = 2.31$. To the contrary, the lower bound elasticities of substitution, $\theta^{lb} = 0.86$, produces a decrease by 11.40% corresponding to a transfer factor of $T_{CV}(0, CV_{GLO}^Y; \theta^{lb}) = 0.89$.

Figure 3 illustrates how mean WTP for global biodiversity conservation changes with the income growth rate or the growth rate of environmental goods. Mean WTP decreases with income growth for substitutes, but increases for complements (Proposition 3). For a hypothetical reduction of the currently expected global income growth rate by half, mean WTP would increase by 192.83% for a substitutability of $\theta^{mean} = 2.31$ and decrease by 36.54% for a substitutability of $\theta^{lb} = 0.86$. When over time more manufactured consumption goods become available, environmental goods lose in value if they
Figure 3: Effect of global per-capita income growth rate, \( g_Y \) (left side) or growth rate of non-use environmental goods, \( g_E \) (right side) on the present value mean WTP for a one percent increase in global non-use environmental goods measured as a constant payment fraction, \( WTP_{CPF,AE} \), for different degrees of substitutability between the consumption and the environmental good, \( \theta \). Grey colour indicates parameter combinations that do not meet the corresponding growth path condition (Eq. (7a)).

are relatively easy to substitute. However, when environmental goods derived from biodiversity complement manufactured goods in consumption, an increase in manufactured goods reinforces the value of environmental goods.

To the contrary mean WTP increases with the growth rate of the environmental good for substitutes, but decreases for complements (Proposition 4). For the non-use environmental goods from global biodiversity, a hypothetically slowing down of the loss rate by half will increase mean WTP by 11.72% for substitutes, \( \theta^{\text{mean}} = 2.31 \), and decrease mean WTP by 3.97% for complements, \( \theta^{\text{lb}} = 0.86 \). Comparing how the loss rate of environmental goods from biodiversity or the income growth rate affect mean WTP for biodiversity conservation, it is apparent that the effect of income growth is relatively stronger.
Next, we study whether the novel structural benefit transfer factors for differences in the growth rates and market interest rates (see Proposition 5) lead to notable WTP adjustments. Specifically, we hypothetically transfer mean WTP elicited at the mean of empirical estimates to a site characterised with the lower or upper bound parameters within the empirically plausible range depicted in Table 2.

First, we turn to the transfer factor for differences in the growth rate of the environmental good $T_{gE}(dE)$ (Eq. (38)). Figure 4 displays the required adjustment when transferring mean WTP from a study site with the global average growth rate of non-use ecosystem services, $g_E^{\text{mean}} = -0.0052$, to a policy site with a growth rate within the range of different non-use ecosystem services’ global growth rates estimated by Baumgärtner et al. (2015). Applying environmental values elicited at a study site with $g_{E}^{\text{study}} := g_{E}^{\text{mean}}$ at a policy site with a higher growth rate of the environmental good, e.g. where the loss of biodiversity is at a lower rate, equal to $g_{E}^{\text{ub}} = -0.0008$ would require an upward adjustment by $21.58\%$ corresponding to a transfer factor $T_{gE}(dE)(g_{E}^{\text{ub}}, g_{E}^{\text{mean}}, \theta^{\text{mean}}) = 1.2158$. To the contrary, for a transfer to a policy context with a rate of biodiversity loss of $g_{E}^{\text{lb}} = -0.0128$ WTP-estimates would need to be lowered by $23.51\%$, i.e. be adjusted by the factor $T_{gE}(dE)(g_{E}^{\text{lb}}, g_{E}^{\text{mean}}, \theta^{\text{mean}}) = 0.7649$. Again, it is evident that the to be employed transfer factors depend on the substitutability between the environmental good in question and human-made consumption goods. A higher degree of substitutability would reinforce these required adjustments, $T_{gE}(dE)(g_{E}^{\text{ub}}, g_{E}^{\text{mean}}, \theta^{\text{ub}}) = 1.3205$ and $T_{gE}(dE)(g_{E}^{\text{lb}}, g_{E}^{\text{mean}}, \theta^{\text{ub}}) = 0.7044$, but complementarity would reverse the direction of the required adjustments, $T_{gE}(dE)(g_{E}^{\text{ub}}, g_{E}^{\text{mean}}, \theta^{\text{lb}}) = 0.9346$ and $T_{gE}(dE)(g_{E}^{\text{lb}}, g_{E}^{\text{mean}}, \theta^{\text{lb}}) = 1.1386$.

Second, adjusting environmental values for differences in income growth rates, $T_{gY,\delta}$ (Eq. (42)), can be substantial (Figure 5). For the depicted transfers we assume that the market interest factor is identical at policy and study site and constant over time, $\delta^{\text{policy}} = \delta^{\text{study}}$, which then cancel out in $T_{gY,\delta}$. Note that depending on $\theta$ we end up in parameter constellations where the growth path condition on $g_Y$ is not fulfilled and the closed-form transfer factor, as given in Eq. (42), cannot be applied any more. In Figure 5 the estimates of the transfer factor for income growth rates are coloured grey at growth rate where the growth path condition does not hold (Eq. (7a)). Applying WTP-estimates
elicited for an income growth rate at the expected global mean, \( g_{Y_{\text{y}}}^{\text{study}} : = g_{Y_{\text{y}}}^{\text{mean}} = 0.017 \), in a policy context where the income growth rate is \( g_{Y_{\text{y}}}^{\text{lb}} = 0.001 \) would imply a transfer factor of \( T_{g_{Y_{\delta}}} (g_{Y_{\text{lb}}}^{\text{lb}}, g_{Y_{\text{y}}}^{\text{mean}}; \theta_{\text{mean}}^{\text{mean}}) = 3.747 \). The direction of adjustment is reversed for complements requiring a downward adjustment with \( T_{g_{Y_{\delta}}} (g_{Y_{\text{lb}}}^{\text{lb}}, g_{Y_{\text{y}}}^{\text{mean}}; \theta_{\text{lb}}^{\text{mean}}) = 0.438 \). For the upper bound substitutability estimate at \( g_{Y_{\text{y}}}^{\text{mean}} \) the growth path condition is not meet and we hence cannot apply the transfer factor. The required adjustments \( T_{g_{Y_{\delta}}} \) are even more pronounced, when applying WTP-estimates in contexts with higher income growth equal to the maximal expected rate, \( g_{Y_{\text{y}}}^{\text{ub}} = 0.05 \). However, for the parameter constellation in this empirical application for \( \theta_{\text{mean}}^{\text{mean}} \) and \( \theta_{\text{ub}}^{\text{mean}} \) the growth path conditions are not meet at \( g_{Y_{\text{y}}}^{\text{ub}} = 0.05 \). The maximum value for applying \( T_{g_{Y_{\delta}}} \) is for \( \theta_{\text{mean}}^{\text{mean}} = 2.31 \) at \( g_{Y_{\text{y}}}^{\text{max}} = 2.31 \) = 0.0197 where the transfer factor approaches zero, while it is generally uncritical for complements. For \( \theta_{\text{ub}}^{\text{mean}} = 0.86 \) applying WTP-estimates from a site characterised by the mean expected income growth rate in a context characterised by the maximal expected rate implies a transfer factor of \( T_{g_{Y_{\delta}}} (g_{Y_{\text{ub}}}^{\text{ub}}, g_{Y_{\text{y}}}^{\text{mean}}; \theta_{\text{ub}}^{\text{lb}}) = 8.763 \).
Figure 5: Transfer factor to adjust mean WTP for a one percent increase in the level of the environmental good from a study site with an income growth rate of $g_{Y}^{\text{study}} = 0.017$ to the income growth rate at a policy site. Colours depict different degrees of substitutability between manufactured consumption goods and the environmental good, $\theta$. Grey colour indicates parameter combinations that do not meet the corresponding growth path condition (Eq. (7a)).

Third, Figure 6 depicts again the transfer factor $T_{g_{Y}, \delta}$ but this time for differences in the market interest rate between a study and a policy site. For illustration the market interest rate is kept constant over time at both sites, $\delta_{r} = \delta \forall \tau$, and the income growth rate at policy and study site are identical and equal to the global average, $g_{Y}^{\text{study}} = g_{Y}^{\text{policy}} = g_{Y}^{\text{mean}}$. For identical growth rates at both sites the first factor in $\mathcal{T}_{g_{Y}, \delta}$ reduces to one and hence the entire transfer factor does not depend on the elasticity of substitution, $\theta$, any more. It shows that differences in market interest rates within the range expected by international experts lead to substantial WTP adjustments: For a hypothetical transfer of mean WTP elicited at $\delta^{\text{mean}} = 0.977$ to a policy site with $\delta^{\text{ub}} = 1$
the required adjustment would be \( T_{gY, \delta}(g_{Y_{\text{mean}}}, \delta_{\text{lb}}, g_{Y_{\text{mean}}}, \delta_{\text{mean}}) = 3.574 \) and thus increase mean WTP by 257.41%. To the contrary a hypothetical transfer to the lower bound forecasted market interest factor, \( \delta_{\text{lb}} = 0.94 \), i.e. a situation with a high market interest rate, would imply a transfer factor of \( T_{gY, \delta}(g_{Y_{\text{mean}}}, \delta_{\text{lb}}, g_{Y_{\text{mean}}}, \delta_{\text{mean}}) = 0.292 \). Moreover, we see that the required adjustment of mean WTP in absolute terms, i.e. \( |T_{gY, \delta} - 1| \), is larger for higher levels of the common income growth rate at study and policy site.
6 Discussion

In this section we discuss several critical assumption that limit the generality of our analysis. Among others, these encompass (i) the simple proportional mapping between natural capital and environmental services, (ii) the exponential growth or decline of income as well as environmental services derived from natural capital, (iii) the purely self-regarding dynastic household, (iv) the direct relationship between the intertemporal elasticity of substitution and the inverse of the elasticity of substitution between consumption goods and environmental goods, (v) the representative household setting, (vi) the intragenerational (spatial) distribution of environmental goods, as well as uncertainty about growth rates and model parameters.

First, we considered a simple proportional mapping between natural capital and environmental good and services. Certainly, the mapping of different forms of natural capital into the services it provides are multifaceted (Fenichel and Abbott 2014). At this stage, it is therefore clear that our analysis is only relevant for those cases of non-consumptive environmental services that may be reasonably described by this simplification. In particular, our work focusses on non-use services derived from natural capital for which WTP information is crucial for public policy.

Second, as exemplary paths for the development of income, respectively consumption, and environmental services derived from natural capital we have considered exponential growth or decline. While this is a prominent case in the long-term analysis of environmental-economic problems (e.g. Baumgärtner et al. 2015, Hoel and Sterner 2007), there may be many different relevant growth or decline paths. As non-market valuation studies often do not specify the exact time path of the evolution of natural capital or environmental goods and services, we leave an analysis of other types of growth dynamics that may be relevant for natural capital valuation for future research.

Third, our analysis assumes purely self-regarding dynastic households. Yet, there may also be behavioral responses to income inequality within and accross generations.

\footnote{We derive this case for income as the result of a balanced growth path in an endogenous growth model in Appendix A.1.}
These may include relative consumption concerns (e.g. Johansson-Stenman and Sterner 2015) or variants of inequality aversion. Again, we leave these extensions to future work.

Fourth, we made a rather strong assumption of a direct relationship between the inverse of the intertemporal elasticity of substitution and the inverse of the elasticity of substitution between consumption goods and environmental goods, \( \eta = 1/\theta \), to be able to derive a closed-form solution of the intertemporal utility function. This assumption follows theoretic work by Quaas and Bröcker (2016), who build a solvable analytic climate-economy model that extends upon the previous Cobb-Douglas cases in the literature (cf. Golosov et al. 2014). In our central calibration, \( 1/\theta \) equals 0.43. This is considered a rather low value of \( \eta \) by most experts (Drupp et al. 2018a). In subsequent versions of the paper, we will try to relax this assumption via simulations.

Fifth, we have assumed that dynastic households are identical except that they differ in initial income. Yet, households may have heterogenous preferences and may also differ not only in their initial income but may also face different income or consumption growth rates. For example, recent empirical evidence from the World Inequality Report (Alvaredo et al. 2018) shows that the growth rate of income differs substantially over income groups. Expert forecasts on consumption growth rates also differ substantially, but appear to be roughly normally distributed. Different growth rates of income give rise to convergence or divergence of income and hence a change in relative income inequality over time. In subsequent versions of the paper, we will try to relax the assumption of equal growth rates for all households also via simulations. With respect to heterogenous preferences, Baumgärtner et al. (2017, Appendix A.12) have already shown in a static context how key results can be extend to feature a normal distribution of preference parameters for the utility weight of environmental goods, \( \alpha \), as well as for the degree of substitutability, \( \theta \). We expect that similar extensions could be made—perhaps under stronger assumptions—in an intertemporal context such as we consider in the present paper. Furthermore, there is a recent literature on heterogeneity in pure rates of time preference (Gollier and Zeckhauser 2005, Millner 2016). The emerging body of literature tends to suggest that the societal discount rate falls over time in the presence of such heterogeneities. In our setting, this may imply that over time the WTP of the more
patient households will dominate. We leave such an extension to heterogeneous discount rates to future work.

Sixth, we have restricted our analysis to the case of pure public environmental goods. While this is a reasonable representation for several important goods and services humans derive from natural capital, such as the existence value of biodiversity studied in our application, there are certainly several environmental goods that vary spatially. The provision of these locally public environmental goods will frequently be correlated with income. For instance, Lee and Lin (2018) show for US metropolitan areas that neighbourhoods close to environmental amenities, such as hills or coastlines, have persisted a high level of income since 1880, and that a heterogeneous endowment with environmental amenities shapes the spatial distribution of incomes. An extension of the model from Baumgärtner et al. (2017) to local public goods is developed in Meya (2018). It shows that for local public goods the effect of income inequality on mean WTP in generally also depends on their provision is correlated with income, but that the main results of Baumgärtner et al. (2017) on how income inequality affects mean WTP generalizes to local public goods being distributed independently of income. We leave an extension of our analysis to a heterogeneous endowment with environmental goods from natural capital and how this distribution evolves over time for future research.

Finally, we have considered a deterministic setting throughout. However, when it comes to issues of intertemporal distribution we must recognize that the world is full of uncertainties. Besides parameter uncertainty, this applies in particular to uncertainty about the growth rates of income or consumption and of environmental goods derived from natural capital. There is a large body of literature on discounting in the presence of uncertainty about baseline growth (e.g. Gollier 2002, 2008). Gollier (2010) considers uncertainty about the growth rate of environmental goods. More recently, Gollier (2017) analyzes how uncertainty about the elasticity of substitution interacts with other forms of uncertainty about growth rates.
7 Conclusion

We have studied how the intra- and intergenerational distribution of income affects the intertemporal valuation of natural capital. To this end we developed a model in which income is distributed unevenly among otherwise identical households, who have constant-elasticity-of-substitution preferences and whose intertemporal elasticity of substitution is the inverse of the elasticity of substitution between environmental goods and manufactured consumption goods.

We find that (i) societal WTP for a marginal increase in the stock or the growth rate of natural capital increases with society’s mean income; (ii) societal WTP for a marginal increase in the stock or the growth rate of natural capital decreases (increases) with society’s income inequality at the time of the valuation if and only if natural capital is a substitute (complement) to manufactured consumption goods; (iii) societal WTP for a marginal increase in stock of natural capital increases with income growth for the case of Cobb-Douglas and complements, but might increase for substitutes; (iv) societal WTP for a marginal increase in stock of natural capita increases (decreases) with the growth rate of environmental goods if and only if natural capital is a substitute (complement) to manufactured goods. Moreover, we derive closed-form adjustment factors for differences in the initial income distribution, growth rates and interest rates. Note that our findings are not confined to environmental public goods, but hold more generally for the valuation of public goods, such as culture, knowledge, open access journals or national security.

Our results are relevant in several respects: First, for benefit transfer in the context of natural capital accounting. Most countries of the world committed them self to mainstream the value of biodiversity in decision making and to integrate biodiversity in national accounts (Convention on Biological Diversity, Aichi Target 2). For instance, EU member states agreed to ”promote the integration of these [economic values of ecosystems and their services] into accounting and reporting systems at EU and national level by 2020” (European Commission 2011, p.15) as part of the EU Biodiversity Strategy (Target 2, Action 5). Approaches to account for natural capital and ecosystem services
in monetary units, usually draw on a set of environmental values and scale these up by means of benefit transfer. As the value of natural capital in accounting systems is defined as the net present values of future ecosystem flows (Obst et al. 2016), the need to apply benefit transfer methods becomes even more evident implying to estimate future flows based today’s valuation studies. Consequently, within the revision process of the *System of Environmental-Economic Accounting – Experimental Ecosystem Accounting* its is emphasised that ”[g]enerally, it will be necessary to apply benefit transfer methods” (United Nations 2017, p.102) for natural capital accounting. However, so far there is limited theory-based guidance for benefit transfer (Bateman et al. 2011) and several international government bodies call for improving benefit transfer methods to enable more accurate ecosystem service and national capital accounting (United Nations et al. 2014). Here we derive to the best of our knowledge for the first time theory-based transfer factors in a dynamic context, which is necessary for a sound accounting of for natural capital. In particular, we show that adjustment can be done by exploiting information on the income distribution at the time of valuation and develop novel factors to control for expected income growth, rates of environmental degradation and interest rates.

Second, these adjustment factors can be used for sustainability policies on natural capital that are concerned with both efficiency and equity by employing equity-adjusted aggregate WTP in environmental cost-benefit analysis (Drupp et al. 2018b).

Finally, our results hold implications for the economic valuation of natural capital. In particular, primary valuation studies should pay attention to the income distribution in the process of aggregating WTPs and report the necessary data to enable a more sophisticated natural capital accounting and determination of their distributional effects.
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Appendix

A.1 Endogenizing the growth and interest rates in a general equilibrium endogenous growth model

The constant growth rate of income, $g_Y$, considered in this paper can be derived as the balanced growth path outcome of an endogenous growth model. To demonstrate this, consider a model with product innovation. Output $Y_t$ is produced by means of labor $L$ and a mass $M_t$ of different types $i$ of machines with input quantities $q_t(i)$, according to the constant-returns-to-scale production function

$$Y_t = \frac{1}{\varphi} L^{1-\varphi} \int_0^{M_t} q_t(i)^{\varphi} \, di,$$

(A.44)

where $\varphi$ is the output elasticity of machinery input.

We normalize labor input to one, $L = 1$, based on the assumption that each of the $n$ households inelastically supplies $1/n$ units of labor. We choose the final output good as the numeraire. Machines fully depreciate after one period of use.

Using $p_t(i)$ to denote the price of a machine of type $i$, input demand by competitive firms in the final goods sector is given by the condition that the value of the marginal product of this machine is equal to its price, i.e.

$$q_t(i)^{\varphi-1} = p_t(i).$$

(A.45)

Blueprints for new types of machines are generated by research and development, which uses the output as the final good as input. Using $Z_t$ to denote the input into R&D at time $t$, the expected mass of new varieties developed is

$$M_{t+1} = M_t + \frac{1}{\Phi} Z_t,$$

(A.46)

with a constant $\Phi > 0$. A firm being successful in R&D becomes the monopolistic supplier for this type of machine. Machines are produced using the final good, such
that one unit of the final good is required to build one unit of a machine. The profit-maximizing price \( p_t(i) \) of a machine of type \( i \) is obtained by maximizing \( p_t(i) q_t(i) - q_t(i) \) subject to (A.45), which yields \( p_t(i) = 1/\varphi \). Using this in (A.45), market equilibrium input of machine type \( i \) is \( q_t(i) = \varphi^{1/(1-\varphi)} \), and total output is

\[
Y_t = M_t \varphi^{\frac{1}{1-\varphi}}. \tag{A.47}
\]

We consider a balanced growth path, such that the interest factor \( \delta_t \) is constant, \( \delta_t = \delta \). The present value of monopoly profits for a firm successful in R&D is \( (p - 1) q/\delta = (1 - \varphi)/(\delta \varphi) \). Under free entry, the expected present value of profits from one dollar spent on R&D must just be equal to this one dollar, i.e.

\[
\Phi \frac{\delta}{1 - \delta} \frac{1 - \varphi}{\varphi} = 1, \tag{A.48}
\]

from which we obtain the interest factor

\[
\delta = \left(1 + \frac{1 - \varphi}{\Phi \varphi}\right)^{-1}. \tag{A.49}
\]

Market equilibrium for final goods implies

\[
Y_t = n C_t + Z_t + \int_0^{M_t} q_t(i) \, di = n C_t + \Phi (M_{t+1} - M_t) + M_t \varphi^{\frac{1}{1-\varphi}}. \tag{A.50}
\]

In a balanced growth path, \( Y_t, C_t, \) and \( M_t \) must thus all grow at the same rate \( g_Y \).

Households choose the intertemporal distribution of consumption such as to maximize

\[
\sum_{t=0}^{\infty} \rho^t \frac{\theta}{\theta - 1} \left( \alpha C_t^{\frac{\theta - 1}{\theta}} + (1 - \alpha) E_t^{\frac{\theta - 1}{\theta}} \right), \tag{A.51}
\]

facing a constant interest factor \( \delta \). The optimal intertemporal distribution of consump-
tion is thus determined by the discrete-time Keynes-Ramsey rule

\[(1 + g_Y)^{\frac{1}{\theta}} = \frac{\rho}{\delta}\] (A.52)

\[\Leftrightarrow g_Y = (\rho/\delta)^{\theta} - 1 = \left(\rho \left(1 + \frac{1 - \phi}{\Phi \varphi}\right)\right)^{\theta} - 1.\] (A.53)

A.2 Derivation of the intertemporal utility function (Eq. (6))

Using (1) in (3), and suppressing the index for household \(i\), gives

\[U(Y_t, E_t) = \sum_{t=0}^{\infty} \rho^t \frac{1}{1 - \eta} \left(\alpha Y_t^{\frac{\varphi - 1}{\varphi}} + (1 - \alpha) E_t^{\frac{\varphi - 1}{\varphi}}\right)^{(1 - \eta)\frac{\theta}{\varphi - 1}}\] (A.54)

\[= \frac{1}{\theta - 1} \left(\sum_{t=0}^{\infty} \rho^t \alpha \left(Y_0 \left(1 + g_Y\right)^t\right)^{\frac{\varphi - 1}{\varphi}} + (1 - \alpha) \sum_{t=0}^{\infty} \rho^t \left(E_0 \left(1 + g_E\right)^t\right)^{\frac{\varphi - 1}{\varphi}}\right)\]

As \(|\rho(1 + g_Y)^{\frac{\varphi - 1}{\varphi}}| < 1\) and \(|\rho(1 + g_E)^{\frac{\varphi - 1}{\varphi}}| < 1\) hold by assumption (Eq. (7a) and Eq. (7b)), the geometric series can be simplified so that one derives the following intertemporal utility function

\[U(U_0, g_Y, E_0, g_E) = \frac{\theta}{\theta - 1} \left(\alpha \frac{Y_0^{\frac{\varphi - 1}{\varphi}}}{1 - \rho(1 + g_Y)^{\frac{\varphi - 1}{\varphi}}} + (1 - \alpha) \frac{E_0^{\frac{\varphi - 1}{\varphi}}}{1 - \rho(1 + g_E)^{\frac{\varphi - 1}{\varphi}}}\right).\] (A.55)

A.3 Derivation of the compensating surplus measured as a single payment, \(x_{SP}\) (Eq. (10))

The compensating surplus, \(x_{SP}\), measured as a single payment in \(t = 0\) is derived by using the instantaneous utility function (Eq. (1)) in the definition of the compensation surplus for a single payment in \(t=0\) (Eq. (8)):
\[
\frac{1}{1-\eta} \left( \alpha (Y_0 - x_{SP})^{\theta^{-1}} + (1 - \alpha) E_0^{\theta^{-1}} \right) + \sum_{t=1}^{\infty} \rho^t \frac{1}{1-\eta} \left( \alpha Y_t^{\theta^{-1}} + (1 - \alpha) E_t^{\theta^{-1}} \right) \\
= \sum_{t=0}^{\infty} \rho^t \frac{1}{1-\eta} \left( \alpha Y_t^{\theta^{-1}} + (1 - \alpha) E_t^{\theta^{-1}} \right)
\]

Assuming \( \eta = \frac{1}{\theta} \) this simplifies to

\[
\alpha (Y_0 - x_{SP})^{\theta^{-1}} + (1 - \alpha) E_0^{\theta^{-1}} + \sum_{t=1}^{\infty} \rho^t \left( \alpha Y_t^{\theta^{-1}} + (1 - \alpha) E_t^{\theta^{-1}} \right) = \sum_{t=1}^{\infty} \rho^t \left( \alpha Y_t^{\theta^{-1}} + (1 - \alpha) E_t^{\theta^{-1}} \right)
\]

\[
\Leftrightarrow \alpha (Y_0 - x_{SP})^{\theta^{-1}} + \sum_{t=1}^{\infty} \rho^t \alpha Y_t^{\theta^{-1}} + \sum_{t=0}^{\infty} \rho^t (1 - \alpha) E_t^{\theta^{-1}} = \sum_{t=1}^{\infty} \rho^t \left( \alpha Y_t^{\theta^{-1}} + (1 - \alpha) E_t^{\theta^{-1}} \right)
\]

\[
\Leftrightarrow \alpha (Y_0 - x_{SP})^{\theta^{-1}} + \sum_{t=1}^{\infty} \rho^t \alpha Y_t^{\theta^{-1}} + \frac{(1 - \alpha) E_0^{\theta^{-1}}}{1 - \rho (1 + g) \frac{\theta^{-1}}{\sigma}} = \frac{\alpha Y_0^{\theta^{-1}}}{1 - \rho (1 + g) \frac{\theta^{-1}}{\sigma}} + \frac{(1 - \alpha) E_0^{\theta^{-1}}}{1 - \rho (1 + gE) \frac{\sigma^{-1}}{\sigma}}.
\]

\[(A.56)\]

Where

\[
\sum_{t=1}^{\infty} \rho^t \alpha Y_t^{\theta^{-1}} = \sum_{t=1}^{\infty} (\rho (1 + g_Y) \frac{\theta^{-1}}{\sigma})^t \alpha Y_t^{\theta^{-1}}
\]

\[
= \sum_{t=0}^{\infty} (\rho (1 + g_Y) \frac{\theta^{-1}}{\sigma})^t \alpha Y_t^{\theta^{-1}} - \left( \rho (1 + g_Y) \frac{\theta^{-1}}{\sigma} \right)^0 \alpha Y_0^{\theta^{-1}}
\]

\[
= \frac{\alpha Y_0^{\theta^{-1}}}{1 - \rho (1 + g_Y) \frac{\theta^{-1}}{\sigma}} - \alpha Y_t^{\theta^{-1}}
\]

\[
= \frac{\left(1 - 1 - \rho (1 + g_Y) \frac{\theta^{-1}}{\sigma}\right) \alpha Y_0^{\theta^{-1}}}{1 - \rho (1 + g_Y) \frac{\sigma^{-1}}{\sigma}}
\]

\[
= \frac{\rho (1 + g_Y) \frac{\theta^{-1}}{\sigma} \alpha Y_0^{\theta^{-1}}}{1 - \rho (1 + g_Y) \frac{\sigma^{-1}}{\sigma}}.
\]
so that Eq. (A.56) becomes

$$\alpha (Y_0 - x_{SP})^{\frac{\sigma - 1}{\sigma}} + \frac{\alpha Y_0^{\frac{\sigma - 1}{\sigma}} \rho (1 + g_Y)^{\frac{\sigma - 1}{\sigma}}}{1 - \rho (1 + g_Y)^{\frac{\sigma - 1}{\sigma}}} + \frac{(1 - \alpha) E_0^{\frac{\sigma - 1}{\sigma}}}{1 - \rho (1 + g_E)^{\frac{\sigma - 1}{\sigma}}} = \frac{\alpha Y_0^{\frac{\sigma - 1}{\sigma}}}{1 - \rho (1 + g_Y)^{\frac{\sigma - 1}{\sigma}}} + \frac{(1 - \alpha) E_0^{\frac{\sigma - 1}{\sigma}}}{1 - \rho (1 + g_E)^{\frac{\sigma - 1}{\sigma}}}.$$  (A.57)

On the left side of this equation is the intertemporal utility (in present value) from consumption of manufactured goods and environmental goods and services spitted in two terms: The first term represents the utility from consumption of manufactured goods in period $t = 0$, for which consumption $Y_0$ is reduced by the one-time payed compensating surplus $x_{SP}$. The second term represents the utility from consumption of manufactured goods starting in period $t = 1$ until the infinite future, for which consumption $Y_0 (1 + g_Y)$ increases with the constant growth rate $g_Y$. Finally, the third term is the share of utility from the consumption of manufactured goods, which decrease from $E_0'$ at the constant rate $g_E$. The right side of this equation is the the present value of the intertemporal utility from the stream of consumption, growing from $Y_0$ at the constant rate $g_Y$, and the stream of environmental goods and services, that decreases from $E_0$ by the constant rate $g_E$.  

This can be reformulated for the compensating surplus, $x_{SP}$, as follows

$$\alpha (Y_0 - x_{SP})^{\frac{\sigma - 1}{\sigma}} + \frac{(1 - \alpha) E_0^{\frac{\sigma - 1}{\sigma}}}{1 - \rho (1 + g_E)^{\frac{\sigma - 1}{\sigma}}} = \frac{\alpha Y_0^{\frac{\sigma - 1}{\sigma}} (1 - \rho (1 + g_Y)^{\frac{\sigma - 1}{\sigma}})}{1 - \rho (1 + g_E)^{\frac{\sigma - 1}{\sigma}}} + \frac{(1 - \alpha) E_0^{\frac{\sigma - 1}{\sigma}}}{1 - \rho (1 + g_E)^{\frac{\sigma - 1}{\sigma}}}.$$
\[(Y_0 - x_{SP})^{\frac{\theta - 1}{\theta}} = Y_0^{\frac{\theta - 1}{\theta}} + \frac{\frac{1-\alpha}{\alpha} E_0^{\frac{\theta - 1}{\theta}}}{1 - \rho (1 + g_E)^{\frac{\theta - 1}{\theta}}} - \frac{\frac{1-\alpha}{\alpha} E_0^{\frac{\theta - 1}{\theta}}}{1 - \rho (1 + g_E')^{\frac{\theta - 1}{\theta}}} \quad (A.58)\]

\[x_{SP} = Y_0 - \left( Y_0^{\frac{\theta - 1}{\theta}} + \frac{\frac{1-\alpha}{\alpha} E_0^{\frac{\theta - 1}{\theta}}}{1 - \rho (1 + g_E)^{\frac{\theta - 1}{\theta}}} - \frac{\frac{1-\alpha}{\alpha} E_0^{\frac{\theta - 1}{\theta}}}{1 - \rho (1 + g_E')^{\frac{\theta - 1}{\theta}}} \right)^{\frac{\theta}{\theta - 1}}\]

\[= Y_0 - \left( Y_0^{\frac{\theta - 1}{\theta}} \left( 1 + Y_0^{\frac{1}{\theta}} \left( \frac{\frac{1-\alpha}{\alpha} E_0^{\frac{\theta - 1}{\theta}}}{1 - \rho (1 + g_E)^{\frac{\theta - 1}{\theta}}} - \frac{\frac{1-\alpha}{\alpha} E_0^{\frac{\theta - 1}{\theta}}}{1 - \rho (1 + g_E')^{\frac{\theta - 1}{\theta}}} \right) \right)^{\frac{\theta}{\theta - 1}} \right)\]

\[= Y_0 \left( 1 - Y_0^{\frac{1}{\theta}} \left( \frac{\frac{1-\alpha}{\alpha} E_0^{\frac{\theta - 1}{\theta}}}{1 - \rho (1 + g_E)^{\frac{\theta - 1}{\theta}}} - \frac{\frac{1-\alpha}{\alpha} E_0^{\frac{\theta - 1}{\theta}}}{1 - \rho (1 + g_E')^{\frac{\theta - 1}{\theta}}} \right)^{\frac{\theta}{\theta - 1}} \right). \quad (A.59)\]

### A.4 Derivation of the compensating surplus measured as a constant fraction, \(x_{CPF}\) (Eq. (11))

Compensating surplus, \(x_{CPF}\), measured as a constant fraction of consumption (Eq. (9)) for the intertemporal utility function specified Eq. (6) is given as

\[U ((1 - x_{CPF})Y_0, g_Y, E_0, g_E) = U (Y_0, g_Y, E_0, g_E) \quad (A.60)\]

\[\frac{\theta}{\theta - 1} \left( \frac{\alpha (1 - x_{CPF})^{\frac{\theta - 1}{\theta}} Y_0^{\frac{\theta - 1}{\theta}}}{1 - \rho (1 + g_E)^{\frac{\theta - 1}{\theta}}} + \frac{(1 - \alpha) E_0^{\frac{\theta - 1}{\theta}}}{1 - \rho (1 + g_E')^{\frac{\theta - 1}{\theta}}} \right) = \frac{\theta}{\theta - 1} \left( \frac{\alpha Y_0^{\frac{\theta - 1}{\theta}}}{1 - \rho (1 + g_Y)^{\frac{\theta - 1}{\theta}}} + \frac{(1 - \alpha) E_0^{\frac{\theta - 1}{\theta}}}{1 - \rho (1 + g_E')^{\frac{\theta - 1}{\theta}}} \right) \quad (A.61)\]

\[\frac{\alpha (1 - x_{CPF})^{\frac{\theta - 1}{\theta}} Y_0^{\frac{\theta - 1}{\theta}}}{1 - \rho (1 + g_Y)^{\frac{\theta - 1}{\theta}}} + \frac{(1 - \alpha) E_0^{\frac{\theta - 1}{\theta}}}{1 - \rho (1 + g_E')^{\frac{\theta - 1}{\theta}}} = \frac{\alpha Y_0^{\frac{\theta - 1}{\theta}}}{1 - \rho (1 + g_Y)^{\frac{\theta - 1}{\theta}}} + \frac{(1 - \alpha) E_0^{\frac{\theta - 1}{\theta}}}{1 - \rho (1 + g_E')^{\frac{\theta - 1}{\theta}}} \quad (A.62)\]
\[
\frac{\alpha \left(1 - (1 - x_{\text{CPF}})^{\frac{\theta - 1}{\theta}}\right) Y_0^{\frac{\theta - 1}{\theta}}}{1 - \rho \left(1 + g_Y\right)^{\frac{\theta - 1}{\theta}}} = \frac{(1 - \alpha) E_0^{\frac{\theta - 1}{\theta}}}{1 - \rho \left(1 + g_Y\right)^{\frac{\theta - 1}{\theta}}} - \frac{(1 - \alpha) E_0^{\frac{\theta - 1}{\theta}}}{1 - \rho \left(1 + g_E\right)^{\frac{\theta - 1}{\theta}}}
\]

\[
x_{\text{CPF}} = 1 - \left(1 - \frac{1 - \rho \left(1 + g_Y\right)^{\frac{\theta - 1}{\theta}}}{\alpha Y_0^{\frac{\theta - 1}{\theta}}} \left(\frac{(1 - \alpha) E_0^{\frac{\theta - 1}{\theta}}}{1 - \rho \left(1 + g_Y\right)^{\frac{\theta - 1}{\theta}}} - \frac{(1 - \alpha) E_0^{\frac{\theta - 1}{\theta}}}{1 - \rho \left(1 + g_E\right)^{\frac{\theta - 1}{\theta}}}\right) \right)^{\frac{\theta}{\theta - 1}}
\]

(A.63)

### A.5 Derivation of the WTP for a marginal change in the initial environmental good, WTP\textsubscript{SP,\text{dE}} (Eq. (12))

Assuming \(g_E = g'_E\) and \(E'_0 = E_0 + dE\) in Eq. (A.58) we can consider the WTP at \(t = 0\) for a marginal change in the initial environmental good

\[
(Y_0 - \text{WTP}_{\text{SP,\text{dE}}}^{\frac{\theta - 1}{\theta}})^{\frac{\theta - 1}{\theta}} = Y_0^{\frac{\theta - 1}{\theta}} + \frac{1 - \alpha}{\alpha} \frac{E_0^{\frac{\theta - 1}{\theta}}}{1 - \rho \left(1 + g_E\right)^{\frac{\theta - 1}{\theta}}} - \frac{1 - \alpha}{\alpha} \frac{(E_0 + dE)^{\frac{\theta - 1}{\theta}}}{1 - \rho \left(1 + g_E\right)^{\frac{\theta - 1}{\theta}}}
\]

(A.64)

\[
= Y_0^{\frac{\theta - 1}{\theta}} + \frac{1 - \alpha}{\alpha} \frac{E_0^{\frac{\theta - 1}{\theta}} - (E_0 + dE)^{\frac{\theta - 1}{\theta}}}{1 - \rho \left(1 + g_E\right)^{\frac{\theta - 1}{\theta}}}
\]

(A.65)

Using first degree Taylor expansion evaluated at \(\text{WTP}_{\text{SP,\text{dE}}} = 0\) we approximate \((Y_0 - \text{WTP}_{\text{SP,\text{dE}}}^{\frac{\theta - 1}{\theta}})^{\frac{\theta - 1}{\theta}} \approx Y_0^{\frac{\theta - 1}{\theta}} - \frac{\theta - 1}{\theta} Y_0^{-1/\theta} \text{WTP}_{\text{SP,\text{dE}}}\) and again using first degree Taylor expansion evaluated at \(dE = 0\) we approximate \((E_0 + dE)^{\frac{\theta - 1}{\theta}} \approx E_0^{\frac{\theta - 1}{\theta}} + \frac{\theta - 1}{\theta} E_0^{-1/\theta} dE\).

This gives

\[
Y_0^{\frac{\theta - 1}{\theta}} - \frac{\theta - 1}{\theta} Y_0^{-1/\theta} \text{WTP}_{\text{SP,\text{dE}}} = Y_0^{\frac{\theta - 1}{\theta}} + \frac{1 - \alpha}{\alpha} \frac{E_0^{\frac{\theta - 1}{\theta}} - E_0^{\frac{\theta - 1}{\theta}} - \frac{\theta - 1}{\theta} E_0^{-1/\theta} dE}{1 - \rho \left(1 + g_E\right)^{\frac{\theta - 1}{\theta}}}
\]

(A.66)

\[
Y_0^{-1/\theta} \text{WTP}_{\text{SP,\text{dE}}} = \frac{1 - \alpha}{\alpha} \frac{E_0^{-1/\theta} dE}{1 - \rho \left(1 + g_E\right)^{\frac{\theta - 1}{\theta}}}
\]

(A.67)

\[
\text{WTP}_{\text{SP,\text{dE}}} = \frac{1 - \alpha}{\alpha} \frac{Y_0^{1/\theta} E_0^{-1/\theta}}{1 - \rho \left(1 + g_E\right)^{\frac{\theta - 1}{\theta}}} dE.
\]

(A.68)
A.6 Derivation of WTP as a single payment in $t=0$ for a marginal change in the growth rate, $\text{WTP}_{SP,dg_E}$ (Eq. (13))

Assuming $g'_E = g_E + dg_E$ and $E_0 = E'_0$ in Eq. (A.58) we can consider the WTP, $\text{WTP}_{SP,dg_E}$, at $t = 0$ for a marginal change of the growth rate of environmental goods.

\[
(Y_0 - \text{WTP}_{SP,dg_E})^{\frac{\theta-1}{\theta}} = Y_0^{\frac{\theta-1}{\theta}} + \frac{1-\alpha}{\alpha} E_0^{\frac{\theta-1}{\theta}} - \frac{1-\alpha}{\alpha} E_0^{\frac{\theta-1}{\theta}} \left( \frac{1}{1 - \rho (1 + g_E)^{\frac{\theta-1}{\theta}}} - \frac{1}{1 - \rho (1 + g_E + dg_E)^{\frac{\theta-1}{\theta}}} \right)
\]

Conducting a first degree Taylor expansion for $f(dg_E) = \frac{1}{1 - \rho (1 + g_E + dg_E)^{\frac{\theta-1}{\theta}}}$ at $dg_E = 0$ yields

\[
\frac{1}{1 - \rho (1 + g_E + dg_E)^{\frac{\theta-1}{\theta}}} \approx \frac{1}{1 - \rho (1 + g_E)^{\frac{\theta-1}{\theta}}} + \frac{\theta-1}{\theta} \rho (1 + g_E)^{-1/\theta} \left( 1 - \rho (1 + g_E)^{\frac{\theta-1}{\theta}} \right)^{-1/2} dg_E
\]

Using (A.70) and $(Y_0 - \text{WTP}_{SP,dg_E})^{\frac{\theta-1}{\theta}} \approx Y_0^{\frac{\theta-1}{\theta}} - \frac{\theta-1}{\theta} Y_0^{-1/\theta}$ WTP$_{SP,dg_E}$ we get

\[
Y_0^{\frac{\theta-1}{\theta}} - \frac{\theta-1}{\theta} Y_0^{-1/\theta} \text{WTP}_{SP,dg_E} =
\]

\[
Y_0^{\frac{\theta-1}{\theta}} + \frac{1-\alpha}{\alpha} E_0^{\frac{\theta-1}{\theta}} \left( \frac{1}{1 - \rho (1 + g_E)^{\frac{\theta-1}{\theta}}} - \frac{1}{1 - \rho (1 + g_E + dg_E)^{\frac{\theta-1}{\theta}}} \right) - \frac{\theta-1}{\theta} Y_0^{-1/\theta} \text{WTP}_{SP,dg_E}
\]

\[
= - \frac{1-\alpha}{\alpha} E_0^{\frac{\theta-1}{\theta}} \frac{\theta-1}{\theta} \rho (1 + g_E)^{-1/\theta} \left( 1 - \rho (1 + g_E)^{\frac{\theta-1}{\theta}} \right)^{-1/2} dg_E
\]

\[
\text{WTP}_{SP,dg_E} = \frac{1-\alpha}{\alpha} E_0^{\frac{\theta-1}{\theta}} Y_0^{1/\theta} \frac{\rho (1 + g_E)^{-1/\theta}}{\left( 1 - \rho (1 + g_E)^{\frac{\theta-1}{\theta}} \right)^2} dg_E
\]
A.7 Derivation of the WTP for a marginal change in the initial environmental good as a constant fraction, \( \text{WTP}_{\text{CPF}, \text{d}E} \) (Eq. (14))

Assuming \( g_E = g'_E \) and \( E'_0 = E_0 + \text{d}E \) in Eq. (A.62) we can consider the WTP in terms of a constant fraction of consumption for a marginal change in the initial environmental good.

\[
\begin{align*}
\alpha (1 - \text{WTP}_{\text{CPF}, \text{d}E}) \frac{\theta - 1}{\theta} Y_0^{\frac{\theta - 1}{\theta}} + (1 - \alpha)(E_0 + \text{d}E)^{\frac{\theta - 1}{\theta}} &= \frac{\alpha Y_0^{\frac{\theta - 1}{\theta}}}{1 - \rho (1 + gY)^{\frac{\theta - 1}{\theta}}} + \frac{(1 - \alpha) E_0^{\frac{\theta - 1}{\theta}}}{1 - \rho (1 + g_E)^{\frac{\theta - 1}{\theta}}} \\
0 &= \frac{\alpha (1 - \text{WTP}_{\text{CPF}, \text{d}E}) \frac{\theta - 1}{\theta} Y_0^{\frac{\theta - 1}{\theta}} - \alpha Y_0^{\frac{\theta - 1}{\theta}}}{1 - \rho (1 + gY)^{\frac{\theta - 1}{\theta}}} + \frac{(1 - \alpha)(E_0 + \text{d}E)^{\frac{\theta - 1}{\theta}} - (1 - \alpha)E_0^{\frac{\theta - 1}{\theta}}}{1 - \rho (1 + g_E)^{\frac{\theta - 1}{\theta}}}
\end{align*}
\]

Applying Taylor series expansion of degree one at \( \text{WTP}_{\text{CPF}, \text{d}E} = 0 \) and \( \text{d}E = 0 \), respectively, yields the following approximations \( (1 - \text{WTP}_{\text{CPF}, \text{d}E})^{\frac{\theta - 1}{\theta}} \approx 1 + \frac{1 - \theta}{\theta} \text{WTP}_{\text{CPF}, \text{d}E} \) and \( (E_0 + \text{d}E)^{\frac{\theta - 1}{\theta}} \approx E_0^{\frac{\theta - 1}{\theta}} + \frac{\theta - 1}{\theta} E_0^{-1/\theta} \text{d}E \). Using these in the formula above yields

\[
\begin{align*}
0 &= \frac{\alpha}{1 - \rho (1 + gY)^{\frac{\theta - 1}{\theta}}} \text{WTP}_{\text{CPF}, \text{d}E} + \frac{(1 - \alpha) E_0^{-1/\theta}}{1 - \rho (1 + g_E)^{\frac{\theta - 1}{\theta}}} \text{d}E \\
\frac{\alpha Y_0^{\frac{\theta - 1}{\theta}}}{1 - \rho (1 + gY)^{\frac{\theta - 1}{\theta}}} \text{WTP}_{\text{CPF}, \text{d}E} &= \frac{(1 - \alpha) E_0^{-1/\theta}}{1 - \rho (1 + g_E)^{\frac{\theta - 1}{\theta}}} \text{d}E \quad (A.71) \\
\text{WTP}_{\text{CPF}, \text{d}E} &= \frac{1 - \alpha}{\alpha} \frac{1 - \rho (1 + gY)^{\frac{\theta - 1}{\theta}}}{Y_0^{\frac{1 - \theta}{\theta}} E_0^{-1/\theta}} \text{d}E. \quad (A.72)
\end{align*}
\]

A.8 Derivation of the WTP for a marginal change in the growth rate of the environmental good as a constant fraction, \( \text{WTP}_{\text{CPF}, \text{dg}E} \) (Eq. (15))

Assuming \( E_0 = E'_0 \) and \( g'_E = g_E + \text{dg}_E \) in Eq. (A.62) we can consider the WTP, \( \text{WTP}_{\text{CPF}, \text{dg}E} \), in terms of a constant fraction of consumption for a marginal change in
the growth rate of the environmental good

\[
0 = \frac{\alpha (1 - \text{WTP}_{\text{CPF}, \text{d}E}) \theta y^{\frac{\theta-1}{\sigma}} Y_0^{\frac{\theta-1}{\sigma}}}{1 - \rho (1 + gy)^{\frac{\theta-1}{\sigma}}} + \frac{(1 - \alpha) E_0^{\frac{\sigma-1}{\theta}}}{1 - \rho (1 + gE + dgE)^{\frac{\sigma-1}{\theta}}} - \frac{(1 - \alpha) E_0^{\frac{\sigma-1}{\theta}}}{1 - \rho (1 + gY)^{\frac{\sigma-1}{\theta}}} + (1 - \alpha) E_0^{\frac{\sigma-1}{\theta}} \left( \frac{1}{1 - \rho (1 + gE)^{\frac{\theta-1}{\sigma}}} - \frac{1}{1 - \rho (1 + gE + dgE)^{\frac{\theta-1}{\sigma}}} \right).
\]

Applying Taylor series expansion of degree one at \( \text{WTP}_{\text{CPF}, \text{d}E} = 0 \) and \( \text{d}g = 0 \), respectively, yields the following approximations \((1 - \text{WTP}_{\text{CPF}, \text{d}E})^{\frac{\theta-1}{\sigma}} \approx 1 - \frac{\theta-1}{\sigma} \text{WTP}_{\text{CPF}, \text{d}E} \)

and

\[
\frac{1}{1 - \rho (1 + gE + \text{d}gE)^{\frac{\theta-1}{\sigma}}} \approx \frac{1}{1 - \rho (1 + gE)^{\frac{\theta-1}{\sigma}}} + \frac{\frac{\theta-1}{\sigma} \rho (1 + gE)^{-1/\theta}}{\left(1 - \rho (1 + gE)^{\frac{\theta-1}{\sigma}}\right)^2} \text{d}gE.
\]

Using these in the formula above yields

\[
0 = \frac{\alpha Y_0^{\frac{\theta-1}{\sigma}} - \alpha (1 - \frac{\theta-1}{\sigma} \text{WTP}_{\text{CPF}, \text{d}E}) Y_0^{\frac{\theta-1}{\sigma}}}{1 - \rho (1 + gy)^{\frac{\theta-1}{\sigma}}} - (1 - \alpha) E_0^{\frac{\sigma-1}{\theta}} \left( \frac{\rho^{-1}(1 + gE)^{-1/\theta}}{1 - \rho (1 + gE)^{\frac{\theta-1}{\sigma}}} \right) \text{d}gE
\]

\[
0 = \frac{\alpha Y_0^{\frac{\theta-1}{\sigma}} - \alpha (1 - \frac{\theta-1}{\sigma} \text{WTP}_{\text{CPF}, \text{d}E}) Y_0^{\frac{\theta-1}{\sigma}}}{1 - \rho (1 + gy)^{\frac{\theta-1}{\sigma}}} - (1 - \alpha) E_0^{\frac{\sigma-1}{\theta}} \left( \frac{\rho^{-1}(1 + gE)^{-1/\theta}}{1 - \rho (1 + gE)^{\frac{\theta-1}{\sigma}}} \right) \text{d}gE
\]

\[
\text{WTP}_{\text{CPF}, \text{d}E} = (1 - \alpha) E_0^{\frac{\sigma-1}{\theta}} \left( \frac{\rho^{-1}(1 + gE)^{-1/\theta}}{1 - \rho (1 + gE)^{\frac{\theta-1}{\sigma}}} \right) \text{d}gE
\]

\[
\text{WTP}_{\text{CPF}, \text{d}E} = \frac{1 - \alpha \rho (1 + gE)^{-1/\theta} \left( 1 - \rho (1 + gY)^{\frac{\theta-1}{\sigma}} \right)}{\alpha \theta Y_0^{\frac{\theta-1}{\sigma}}} \left( 1 - \rho (1 + gE)^{\frac{\theta-1}{\sigma}} \right)^2 Y_0^{\frac{\theta-1}{\sigma}} E_0^{\frac{\sigma-1}{\theta}} \text{d}gE
\]
A.9 Derivation of mean WTP as a single payment for a marginal change in the level of the environmental good, $\bar{WTP}_{SP,dE}$ (Eq. (17))

The density function of the log-normal distribution of consumption $Y_0$ in $t = 0$ with mean $\mu_{Y_0}$ and standard deviation $\sigma_{Y_0}$ is given by

$$f_{ln}(Y_0; \mu_{Y_0}, \sigma_{Y_0}) = \frac{1}{Y_0 \sqrt{2\pi s^2}} \exp \left( -\frac{(\ln Y_0 - m)^2}{2s^2} \right)$$  \hspace{1cm} (A.73)

with

$$m = \ln \mu_{Y_0} - \frac{1}{2} \ln \left( 1 + \frac{\sigma_{Y_0}^2}{\mu_{Y_0}^2} \right)$$  \hspace{1cm} (A.74)

$$s^2 = \ln \left( 1 + \frac{\sigma_{Y_0}^2}{\mu_{Y_0}^2} \right)$$  \hspace{1cm} (A.75)

Then mean compensating surplus in terms of a single payment at $t = 0$ (Eq. (16))
can be reformulated as

$$\text{WTP}_{SP,dE}(\mu Y_0, \sigma Y_0) = \int_0^\infty f_{\ln}(Y_0; \mu Y_0, \sigma Y_0) \text{WTP}_{SP,dE}(Y_0) \, dY_0$$

\( (12), (A.73) \)

$$= \left[ \frac{1}{\sqrt{2\pi s^2}} \exp \left( -\frac{(\ln Y_0 - m)^2}{2s^2} \right) \frac{1 - \alpha}{\alpha} Y_0^{-1/\theta} \frac{E_0^{-1/\theta} \, dE_0}{1 - \rho (1 + g_E)^{-\theta}} \int_0^\infty Y_0^{1/\theta} \exp \left( -\frac{(\ln Y_0 - m)^2}{2s^2} \right) \, dY_0 \right]$$

$$= \kappa \int_0^\infty \frac{Y_0^{1-\theta}}{\sqrt{2\pi s^2}} \exp \left( -\frac{(\ln Y_0 - m)^2}{2s^2} \right) \, dY_0$$

$$= \kappa \exp \left[ \frac{1}{2\theta^2} s^2 + \frac{1}{\theta} m \right] \frac{1}{2\theta^2} \ln \left( 1 + \frac{\sigma^2 Y_0}{\mu Y_0} \right) + \frac{1}{\theta} \ln \mu Y_0 - \frac{1}{2\theta} \ln \left( 1 + \frac{\sigma^2 Y_0}{\mu Y_0} \right)$$

\( (A.74), (A.75) \)

$$= \kappa \exp \left[ \frac{1 - \theta}{2\theta^2} \ln \left( 1 + \frac{\sigma^2 Y_0}{\mu Y_0} \right) + \frac{1}{\theta} \ln(\mu Y_0) \right]$$

$$= \kappa \exp \left[ \ln \left( 1 + \frac{\sigma^2 Y_0}{\mu Y_0} \right)^{\frac{1-\theta}{2\theta^2}} \right] \exp \left[ \ln(\mu Y_0) \right]$$

$$= \kappa \mu Y_0^{1/\theta} \left( 1 + \frac{\sigma^2 Y_0}{\mu Y_0} \right)^{\frac{1-\theta}{2\theta^2}} \quad (A.76)$$

or, equivalently, for relative inequality in initial consumption \( CV_0 = \frac{\sigma Y_0}{\mu Y_0} \)

$$\text{WTP}_{SP,dE}(\mu Y_0, CV_0) = \kappa \mu Y_0^{1/\theta} \left( 1 + CV_0 \right)^{\frac{1-\theta}{2\theta^2}}. \quad (A.77)$$
A.10 Derivation of the mean WTP as a single payment for a marginal change in the growth rate of the environmental good, $\overline{WTP}_{sp,dgE}$ (Eq. (18))

Analogue to Appendix A.9 we derive the mean WTP for a marginal change in the growth rate:

$$\overline{WTP}_{sp,dgE}(\mu_{Y_0}, \sigma_{Y_0}) = \int_{Y_0}^{\infty} f_{\ln(Y_0; \mu_{Y_0}, \sigma_{Y_0})} WTP_{sp,dgE}(Y_0) dY_0$$

$$= \int_{Y_0}^{\infty} \exp \left( -\frac{(\ln Y_0 - m)^2}{2s^2} \right) \frac{1}{Y_0 \sqrt{2\pi s^2}} \frac{1 - \alpha}{\alpha} E_0 \frac{\rho(1 + g_E)^{-1/\theta}}{(1 - \rho(1 + g_E)^{\frac{s-1}{\sigma}})^2} dY_0$$

$$= \frac{1 - \alpha}{\alpha} E_0 \left( \frac{\rho(1 + g_E)^{-1/\theta}}{(1 - \rho(1 + g_E)^{\frac{s-1}{\sigma}})^2} \right) \overline{WTP}_{sp,dgE}(Y_0)$$

$$= \kappa' \exp \left[ \frac{1}{2 \theta^2} \frac{s^2}{s^2} + \frac{1}{\theta} m \right]$$

$$= \kappa' \mu_{Y_0}^{\frac{1}{\theta}} \left( 1 + \frac{\sigma_{Y_0}^2}{\mu_{Y_0}^2} \right)^{\frac{1-\theta}{2\theta}} \text{ (A.78)}$$

or, equivalently, for relative inequality in initial consumption $CV_{Y_0} = \frac{\sigma_{Y_0}}{\mu_{Y_0}}$

$$\overline{WTP}_{sp,dgE}(\mu_{Y_0}, CV_{Y_0}) = \kappa' \mu_{Y_0}^{\frac{1}{\theta}} \left( 1 + CV_{Y_0}^2 \right)^{\frac{1-\theta}{2\theta}} \text{. (A.79)}$$
A.11 Derivation of mean WTP at time \( t \), \( \overline{\text{WTP}}_{\text{CPF},dE,t} \) (Eq. (20)), and the present value of mean WTP, \( \overline{\text{WTP}}_{\text{CPF},dE} \) (Eq. (22)), for a marginal change in stock and a constant payment fraction

The mean WTP at time \( t \) measured as a constant fraction of consumption is than

\[
\overline{\text{WTP}}_{\text{CPF},dE,t}(\mu_Y, \sigma_Y) = \int_0^\infty f_{\text{ln}}(Y_0; \mu_Y, \sigma_Y) \overline{\text{WTP}}_{\text{CPF},dE}(Y_0) Y_t(Y_0) dY_0
\]

\[
\equiv \int_0^\infty \frac{1}{Y_0 \sqrt{2\pi s^2}} \exp \left( -\frac{(\ln Y_0 - m)^2}{2s^2} \right) \frac{1 - \alpha}{\alpha} \frac{1 - \rho (1 + g_Y)^{\frac{\sigma - 1}{\sigma}}}{1 - \rho (1 + g_E)^{\frac{\sigma - 1}{\sigma}}} E_0^{-1/\theta} Y_0^{1-\theta} dE(1 + g_Y)^t Y_0 dY_0
\]

\[
= \frac{1 - \alpha}{\alpha} \left( 1 - \rho (1 + g_Y)^{\frac{\sigma - 1}{\sigma}} \right) \left( 1 + g_Y \right)^t \int_0^\infty \frac{Y_0^{1-\theta}}{\sqrt{2\pi s^2}} \exp \left( -\frac{(\ln Y_0 - m)^2}{2s^2} \right) dY_0
\]

\[
= \kappa'' \mu_Y \left( 1 + \frac{\sigma_Y^2}{\mu_Y^2} \right) \frac{1-\theta}{2\sigma^2}
\]

Sec.A.9 \( \equiv \kappa'' \mu_Y \left( 1 + \frac{\sigma_Y^2}{\mu_Y^2} \right) \frac{1-\theta}{2\sigma^2} \) (A.80)

and for relative inequality in initial consumption, \( CV_Y = \frac{\sigma_Y}{\mu_Y} \),

\[
\overline{\text{WTP}}_{\text{CPF},dE,(\mu_Y, CV_Y)} = \kappa'' \mu_Y \left( 1 + CV_Y \right)^{1/\theta} \left( 1 + CV_Y^2 \right)^{1-\theta/2\sigma^2}
\]

(A.81)
The associated present value - discounted at market interest rates - is

\[ WTP_{CPF,dE}(\mu_Y, CV_Y) = \sum_{t=0}^{\infty} \left( \prod_{\tau=0}^{t} \delta_{\tau} \right) WTP_{CPF,dE; t}(\mu_Y, CV_Y) \]

\[
= \sum_{t=0}^{\infty} \left( \prod_{\tau=0}^{t} \delta_{\tau} \right) \frac{1 - \frac{\alpha}{\alpha}(1 - \rho(1 + g_Y)^{\frac{\sigma - 1}{\sigma}})}{1 - \rho(1 + g_E)^{\frac{\sigma - 1}{\sigma}}} E_0^{-1/\theta} dE \mu_Y^{1/\theta} (1 + CV_Y^{2})^{\frac{1 - \theta}{2\theta}}
\]

\[
= \frac{1 - \frac{\alpha}{\alpha} - \rho(1 + g_Y)^{\frac{\sigma - 1}{\sigma}}}{1 - \rho(1 + g_E)^{\frac{\sigma - 1}{\sigma}}} E_0^{-1/\theta} dE \sum_{t=0}^{\infty} \left( \prod_{\tau=0}^{t} \delta_{\tau} \right) (1 + g_Y)^t \mu_Y^{1/\theta} (1 + CV_Y^{2})^{\frac{1 - \theta}{2\theta}}
\]

\[
= \kappa'' \mu_Y^{1/\theta} (1 + CV_Y^{2})^{\frac{1 - \theta}{2\theta}}.
\] (A.82)

A.12 Derivation of mean WTP at time \( t \), \( WTP_{CPF,dg_E; t} \) (Eq. (24)), and the present value of mean WTP, \( WTP_{CPF,dg_E} \) (Eq. (24)), for a marginal change in growth rate and constant payment fraction

\[ WTP_{CPF,dg_E; t}(\mu_Y, \sigma_Y) \]

\[
= \int_{0}^{\infty} f_{\ln}(Y_0; \mu_Y, \sigma_Y) WTP_{CPF,dg_E}(Y_0) Y_0 dY_0
\]

\[
= \int_{0}^{\infty} f_{\ln}(Y_0; \mu_Y, \sigma_Y) WTP_{CPF,dg_E}(Y_0) (1 + g_Y)^t Y_0 dY_0
\]

\[
= \int_{0}^{\infty} \frac{1}{Y_0 \sqrt{2\pi s^2}} \exp \left( -\frac{(\ln Y_0 - m)^2}{2s^2} \right) \frac{1 - \alpha}{\alpha} \rho(1 + g_E)^{-1/\theta} \left( 1 - \frac{\rho(1 + g_Y)^{\frac{\sigma - 1}{\sigma}}}{(1 - \rho(1 + g_E)^{\frac{\sigma - 1}{\sigma}})^2} \right)
\]

\[
Y_0^{\frac{1 - \theta}{\sigma}} E_0^{\frac{\theta - 1}{\sigma}} dE (1 + g_Y)^t Y_0 dY_0
\]

\[
= \frac{1 - \frac{\alpha}{\alpha} \rho(1 + g_E)^{-1/\theta} (1 - \frac{\rho(1 + g_Y)^{\frac{\sigma - 1}{\sigma}}}{(1 - \rho(1 + g_E)^{\frac{\sigma - 1}{\sigma}})^2})}{\rho(1 + g_E)^{-1/\theta}} dE (1 + g_Y)^t E_0^{\frac{\theta - 1}{\sigma}} Y_0^{\frac{1 - \theta}{\sigma}} \int_{0}^{\infty} \frac{1}{\sqrt{2\pi s^2}} \exp \left( -\frac{(\ln Y_0 - m)^2}{2s^2} \right) dY_0
\]

\[
= \kappa''' \mu_Y^{1/\theta} \left( 1 + CV_Y^{2} \right)^{\frac{1 - \theta}{2\theta}}
\] (A.83)
and for relative inequality in initial consumption, $CV_{Y_0} = \frac{\sigma_{Y_0}}{\mu_{Y_0}}$, 

$$WTP_{CPF, dgE; \mu_{Y_0}, CV_{Y_0}} = \kappa'''' \frac{1}{\theta} \mu_{Y_0} (1 + CV_{Y_0}^2) \frac{1-\theta}{2\theta^2} \tag{A.84}$$

The associated present value - discounted at market interest rates - is

$$WTP_{CPF, dgE} (\mu_{Y_0}, CV_{Y_0}) = \sum_{t=0}^{\infty} \left( \prod_{\tau=0}^{t} \delta_{\tau} \right) WTP_{CPF, dgE; \mu_{Y_0}, CV_{Y_0}}$$

$$= \kappa'''' \frac{1}{\theta} \mu_{Y_0} (1 + CV_{Y_0}^2) \frac{1-\theta}{2\theta^2} \tag{A.85}$$

**A.13 Proof of Proposition 1**

Differentiating $WTP_{SP,dE} (\mu_{Y_0}, CV_{Y_0})$ (Eq. (17)) with respect to mean consumption, $\mu_{Y_0}$, yields

$$\frac{\partial WTP_{SP,dE} (\mu_{Y_0}, CV_{Y_0})}{\partial \mu_{C0}} = \kappa'''' \frac{1}{\theta} \mu_{Y_0} (1 + CV_{Y_0}^2) \frac{1-\theta}{2\theta^2} \tag{A.86}$$

with $\kappa = \frac{1 - \alpha}{\alpha} \frac{E_0^{-1/\theta} dE}{1 - \rho (1 + g_E)^{\theta-1}}$.

which is strictly greater zero for a marginal increase in the environmental good ($dE > 0$), as $E_0, \mu_{Y_0}, CV_{Y_0}, \theta > 0$, $\alpha \in (0,1)$ and by assumption $\rho (1 + g_E)^{\theta-1} < 1$.

Differentiating $WTP_{SP,dE} (\mu_{Y_0}, CV_{Y_0})$ (Eq. 18) with respect to mean consumption,
μ₀, yields

\[
\frac{\partial \text{WTP}_{SP,dgE}(μ₀, CV₀)}{\partial µ_c} = \frac{1}{1 - \rho(1 + g_E)^{\frac{\theta - 1}{\theta}}} \frac{1 + CV₀^2}{Y₀^{\frac{1 - \theta}{2\theta}}} \frac{\rho(1 + g_E)^{-1/\theta}}{\left(1 - \rho(1 + g_E)^{\frac{\theta - 1}{\theta}}\right)^2} \frac{dg_E}{dE}.
\]

(A.87)

which is strictly greater zero for a marginal increase in the environmental good (dgE > 0), as E₀, μ₀, CV₀, Y₀, ρ > 0, α ∈ (0, 1) and g_E ∈ (-1, 0].

Differentiating \(\text{WTP}_{CPF,dE}(μ₀, CV₀)\) (Eq. (22)) with respect to mean consumption, μ₀, yields

\[
\frac{\partial \text{WTP}_{CPF,dE}(μ₀, CV₀)}{\partial µ_c} = \frac{1}{1 - \rho(1 + g_E)^{\frac{\theta - 1}{\theta}}} \frac{1 + CV₀^2}{Y₀^{\frac{1 - \theta}{2\theta}}} \frac{\rho(1 + g_E)^{-1/\theta}}{\left(1 - \rho(1 + g_E)^{\frac{\theta - 1}{\theta}}\right)^2} \frac{dg_E}{E}
\]

(A.88)

which is strictly greater zero for a marginal increase in the environmental good (dE > 0), as E₀, μ₀, CV₀, θ, ρ > 0, α ∈ (0, 1) and by assumption ρ(1 + g_E)^{\frac{\theta - 1}{\theta}} < 1 and ρ(1 + g_E)^{\frac{\theta - 1}{\theta}} < 1.

Differentiating \(\text{WTP}_{CPF,dgE}(μ₀, CV₀)\) (Eq. (24)) with respect to mean income, μ₀, yields

\[
\frac{\partial \text{WTP}_{CPF,dgE}(μ₀, CV₀)}{\partial µ_c} = \frac{1}{1 - \rho(1 + g_E)^{\frac{\theta - 1}{\theta}}} \frac{1 + CV₀^2}{Y₀^{\frac{1 - \theta}{2\theta}}} \frac{\rho(1 + g_E)^{-1/\theta}}{\left(1 - \rho(1 + g_E)^{\frac{\theta - 1}{\theta}}\right)^2} \frac{dg_E}{E} \frac{dg_E}{E} \left(1 + g_E\right)^{t}
\]

(A.89)

which is strictly greater zero for a marginal increase in the environmental good (dgE > 0), as α ∈ (0, 1), g_E > -1, ρ(1 + g_E)^{\frac{\theta - 1}{\theta}} < 1 and g_E, E₀, CV₀, μ₀, ρ > 0.
A.14 Proof of Proposition 2

Differentiating \( \text{WTP}_{SP, dE}(\mu_Y, CV_Y) \) (Eq. (17)) with respect to relative inequality in consumption, \( CV_Y \), yields

\[
\frac{\partial \text{WTP}_{SP, dE}(\mu_Y, CV_Y)}{\partial CV_Y} = \kappa \frac{1 - \theta}{\theta^2} \mu_Y^{1/\theta} CV_Y \left(1 + CV_Y^2\right)^{-\theta-2\theta^2} \cdot \quad (A.90)
\]

The sign of the derivative is determined by the sign of the factor \( 1 - \theta \), as \( \mu_Y, CV_Y, \kappa > 0 \). It holds that \( 1 - \theta \gtrless 0 \) if and only if \( \theta \gtrless 1 \).

Differentiating \( \text{WTP}_{SP, dg, E}(\mu_Y, CV_Y) \) (Eq. (18)) with respect to income inequality, \( CV_Y \), yields

\[
\frac{\partial \text{WTP}_{SP, dg, E}(\mu_Y, CV_Y)}{\partial CV_Y} = \kappa' \frac{1 - \theta}{\theta^2} \mu_Y^{1/\theta} CV_Y \left(1 + CV_Y^2\right)^{-\theta-2\theta^2} \cdot \quad (A.91)
\]

The sign of the derivative is determined by the sign of the factor \( 1 - \theta \), as \( \mu_Y, CV_Y, \kappa' > 0 \). It holds that \( 1 - \theta \gtrless 0 \) if and only if \( \theta \gtrless 1 \).

Differentiating \( \text{WTP}_{CPF, dE}(\mu_Y, CV_Y) \) (Eq. (22)) with respect to relative inequality in consumption, \( CV_Y \), yields

\[
\frac{\partial \text{WTP}_{CPF, dE}(\mu_Y, CV_Y)}{\partial CV_Y} = \kappa'' \frac{1 - \theta}{\theta^2} \mu_Y^{1/\theta} CV_Y \left(1 + CV_Y^2\right)^{-\theta-2\theta^2} \cdot \quad (A.92)
\]

The sign of the derivative is determined by the sign of the factor \( 1 - \theta \), as \( \mu_Y, CV_Y, \kappa'' > 0 \). It holds that \( 1 - \theta \gtrless 0 \) if and only if \( \theta \gtrless 1 \).

Differentiating \( \text{WTP}_{CPF, dg, E}(\mu_Y, CV_Y) \) (Eq. (24)) with respect to relative inequality in consumption, \( CV_Y \), yields

\[
\frac{\partial \text{WTP}_{CPF, dg, E}(\mu_Y, CV_Y)}{\partial CV_Y} = \kappa''' \frac{1 - \theta}{\theta^2} \mu_Y^{1/\theta} CV_Y \left(1 + CV_Y^2\right)^{-\theta-2\theta^2} \cdot \quad (A.93)
\]

The sign of which is determined by the factor \( 1 - \theta \), as \( \kappa''' > 0 \). It again holds that \( 1 - \theta \gtrless 0 \) if and only if \( \theta \gtrless 1 \).
A.15 Proof of Proposition 3

Assume time-constant market interest factor, i.e. \( \delta_t = \delta \forall t \). Differentiating \( WTP_{CPF,d} \) (Eq. (22)) with respect to the growth rate of income, \( g_Y \), yields

\[
\frac{\partial WTP_{CPF,d}}{\partial g_Y} = K \left( -\frac{\theta - 1}{\theta} \rho (1 + g_Y)^{-1/\theta} \sum_{t=0}^{\infty} \delta^t (1 + g_Y)^t + \left[ 1 - \rho (1 + g_Y)^{2/\theta} \right] \sum_{t=0}^{\infty} t \delta^t (1 + g_Y)^{t-1} \right)
\]

\[
= K \left( -\frac{\theta - 1}{\theta} \rho (1 + g_Y)^{-1/\theta} \sum_{t=0}^{\infty} \delta_t (1 + g_Y)^t + \frac{1 - \rho (1 + g_Y)^{2/\theta}}{1 + g_Y} \sum_{t=0}^{\infty} t \delta_t (1 + g_Y)^t \right)
\]

with \( K := \frac{1 - \alpha}{\alpha} E_0^{-1/\theta} dE \mu_{Y_0}^{1/\theta} (1 + CV_{Y_0}^2)^{1/2\theta} > 0 \).

For \( \delta(1 + g_Y) < 1 \) the geometric series converge and this becomes

\[
\frac{\partial WTP_{CPF,d}}{\partial g_Y} = K \left[ -\frac{\theta - 1}{\theta} \rho (1 + g_Y)^{-1/\theta} \frac{\delta(1 - \delta (1 + g_Y)) + \delta (1 - \rho (1 + g_Y)^{2/\theta})}{(1 - \delta (1 + g_Y))^2} \right]
\]

\[
= K \left( -\frac{\theta - 1}{\theta} \rho (1 + g_Y)^{-1/\theta} \frac{(1 - \delta (1 + g_Y)) + \rho (1 - \delta (1 + g_Y)) + \delta}{(1 - \delta (1 + g_Y))^2} \right)
\]

\[
= K \left( 1 + g_Y \right)^{-1/\theta} \frac{-\rho + \frac{1}{\theta} \rho (1 - \delta (1 + g_Y)) + \delta (1 + g_Y)^{1/\theta}}{(1 - \delta (1 + g_Y))^2}
\]

\[
= K \left( 1 + g_Y \right)^{-1/\theta} \frac{\rho (-1 + \frac{1}{\theta}) (1 - \delta (1 + g_Y)) + \delta (-1 + g_Y)^{1/\theta}}{(1 - \delta (1 + g_Y))^2}
\]

\[
= K \left( 1 + g_Y \right)^{-1/\theta} \frac{1 - \theta}{\theta} \rho (1 - \delta (1 + g_Y)) + \delta (1 + g_Y)^{1/\theta}
\]

\[
\frac{\partial WTP_{CPF,d}}{\partial g_Y} > 0 \quad \text{if} \quad \theta \leq 1.
\]

As \( E_0, \mu_{Y_0}, CV_{Y_0}, \theta, \rho, g_Y, \delta_r > 0, \alpha \in (0, 1), \rho (1 + g_E)^{2/\theta} < 1 \) (Eq. 7b) and as assumed above \( \delta(1 + g_Y) < 1 \), the only term of this expression that can turn negative is \( \frac{1 - \theta}{\theta} \rho (1 - \delta (1 + g_Y)) \) whose sign if fully determined by the factor \( 1 - \theta \). It thus holds that

\[
\frac{\partial WTP_{CPF,d}}{\partial g_Y} > 0 \quad \text{if} \quad \theta \leq 1.
\]
A.16 Conditions for WTP that declines with income growth

Figure 7: The derivative of mean WTP for a constant payment fraction with respect to the growth rate of income and how its sign and magnitude depend on the elasticity of substitution, $\theta$, and the pure time discount factor, $\rho$.

A.17 Proof of Proposition 4

Differentiating $\text{WTP}_{SP,dE}$ (Eq. (17)) with respect to the growth rate of the environmental good, $g_E$, yields

$$\frac{\partial \text{WTP}_{SP,dE}}{\partial g_E} = K' \frac{\theta - 1}{\theta} \rho \frac{(1 + g_E)^{-1/\theta}}{(1 - \rho (1 + g_E)^{\frac{\theta - 1}{\theta}})^2}$$

with $K' := \frac{1 - \alpha}{\alpha} E_0^{-1/\theta} dE \mu^{-1/\theta} Y_0^{1/\theta} (1 + CV_{Y_0}^2)^{\frac{1-\theta}{2\theta}}$. 

60
As $E_0, \mu_Y, CV_Y, \theta, \rho, \alpha \in (0, 1), g_E > -1, \rho(1 + g_E)^{\frac{\theta-1}{\theta}} < 1$ (Eq. 7b) the sign of \( \frac{\partial WTP_{SP, dE}}{\partial g_E} \) is determined by the sign of $\theta - 1$ and it follows directly that

\[
\frac{\partial WTP_{SP, dE}(\mu_Y, CV_Y, g_Y, g_E)}{\partial g_E} \gtrless 0 \quad \text{if and only if} \quad \theta \gtrless 1.
\]

Differentiating $WTP_{CPF, dE}$ (Eq. (22)) with respect to the growth rate of the environmental good, $g_E$, yields

\[
\frac{\partial WTP_{CPF, dE}}{\partial g_E} = K'' \frac{\theta - 1}{\theta} \rho \frac{(1 + g_E)^{-1/\theta}}{(1 - \rho(1 + g_E)^{\frac{\theta-1}{\theta}})^2}
\]

with $K'' := \frac{1 - \alpha}{\alpha} (1 - \rho(1 + g_Y)^{\frac{\theta-1}{\theta}}) E_0^{1/\theta} dE \left[ \sum_{t=0}^{\infty} \left( \prod_{\tau=0}^{t} \delta_{\tau} \right)(1 + g_Y)^t \right] \mu_Y^{1/\theta} (1 + CV_Y^2)^{\frac{1-\theta}{2\theta}}$.

As $E_0, \mu_Y, CV_Y, \theta, g_Y, \delta_\tau > 0, \alpha \in (0, 1), \rho(1 + g_Y)^{\frac{\theta-1}{\theta}} < 1$ (Eq. 7a), $\rho(1 + g_E)^{\frac{\theta-1}{\theta}} < 1$ (Eq. 7b) the sign of \( \frac{\partial WTP_{CPF, dE}}{\partial g_E} \) is determined by the sign of $\theta - 1$ and it follows directly that

\[
\frac{\partial WTP_{CPF, dE}(\mu_Y, CV_Y, g_Y, g_E)}{\partial g_E} \gtrless 0 \quad \text{if and only if} \quad \theta \gtrless 1.
\]

### A.18 Proof of Proposition 5

The transfer function is defined by the quotient of the mean WTPs for a marginal change in the level or the growth rate of the environmental good at the policy site and study site.

The transfer function for mean WTP elicited at study site and policy site as a single payment for a change in the level of the environmental good, $WTP_{SP, dE}(\mu_Y, CV_Y)$
(Eq. (17)), is given as

\[
T_{SP,dE}(\cdot) = \frac{\text{WTP}_{SP,dE}^{\text{policy}}(\mu_{Y_0}^{\text{policy}}, CV_{Y_0}^{\text{policy}})}{\text{WTP}_{SP,dE}^{\text{study}}(\mu_{Y_0}^{\text{study}}, CV_{Y_0}^{\text{study}})}
\]

\[
(17) = \frac{1-\alpha}{\alpha} \frac{E^{\text{policy}^{-1/\theta}}_0 dE^{\text{policy}}}{1-\rho(1+g_E^{\text{policy}})^{-1/\theta}} \frac{\mu_{Y_0}^{\text{policy}}1/\theta (1 + CV_{Y_0}^{\text{policy}2})^{1-\theta}}{1-\rho(1+g_E^{\text{study}})^{-1/\theta}} \frac{E^{\text{study}^{-1/\theta}}_0 dE^{\text{study}}}{1-\rho(1+g_E^{\text{study}})^{-1/\theta}} \frac{\mu_{Y_0}^{\text{study}1/\theta} (1 + CV_{Y_0}^{\text{study}2})^{1-\theta}}{1-\rho(1+g_E^{\text{study}})^{-1/\theta}}
\]

\[
= \left( \frac{E^{\text{policy}}_0}{E^{\text{study}}_0} \right)^{-1/\theta} \frac{dE^{\text{policy}}}{dE^{\text{study}}} \frac{1-\rho(1+g_E^{\text{policy}})^{-1/\theta}}{1-\rho(1+g_E^{\text{study}})^{-1/\theta}} \cdot \left( \frac{\mu_{Y_0}^{\text{policy}1/\theta}}{\mu_{Y_0}^{\text{study}1/\theta}} \right) \cdot \left( \frac{1 + CV_{Y_0}^{\text{policy}2}}{1 + CV_{Y_0}^{\text{study}2}} \right)^{1-\theta} \frac{1-\rho(1+g_E^{\text{policy}})^{-1/\theta}}{1-\rho(1+g_E^{\text{study}})^{-1/\theta}}
\]

The transfer function for mean WTP elicited at study site and policy site as a single payment for a change in the growth rate of the environmental good, \(\text{WTP}_{SP,dgE}(\mu_{Y_0}, CV_{Y_0})\) (Eq. (18)), is given as

\[
T_{SP,dgE}(\cdot) = \frac{\text{WTP}_{CPF,dE}^{\text{policy}}(\mu_{Y_0}^{\text{policy}}, CV_{Y_0}^{\text{policy}})}{\text{WTP}_{CPF,dE}^{\text{study}}(\mu_{Y_0}^{\text{study}}, CV_{Y_0}^{\text{study}})}
\]

\[
(18) = \frac{1-\alpha}{\alpha} \frac{E^{\text{policy}^{-1/\theta}}_0 dE^{\text{policy}}}{1-\rho(1+g_E^{\text{policy}})^{-1/\theta}} dE^{\text{policy}} \mu_{Y_0}^{\text{policy}1/\theta} (1 + CV_{Y_0}^{\text{policy}2})^{1-\theta}}{1-\rho(1+g_E^{\text{study}})^{-1/\theta}} \frac{E^{\text{study}^{-1/\theta}}_0 dE^{\text{study}}}{1-\rho(1+g_E^{\text{study}})^{-1/\theta}} dE^{\text{study}} \mu_{Y_0}^{\text{study}1/\theta} (1 + CV_{Y_0}^{\text{study}2})^{1-\theta}}
\]

\[
= \left( \frac{E^{\text{policy}}_0}{E^{\text{study}}_0} \right)^{\theta-1/\theta} \frac{dE^{\text{policy}}}{dE^{\text{study}}} \frac{\rho(1+g_E^{\text{policy}})^{-1/\theta}}{\rho(1+g_E^{\text{study}})^{-1/\theta}} dE^{\text{policy}} \mu_{Y_0}^{\text{policy}1/\theta} (1 + CV_{Y_0}^{\text{policy}2})^{1-\theta}}{dE^{\text{study}} \mu_{Y_0}^{\text{study}1/\theta} (1 + CV_{Y_0}^{\text{study}2})^{1-\theta}}
\]

\[
\cdot \left( \frac{\mu_{Y_0}^{\text{policy}1/\theta}}{\mu_{Y_0}^{\text{study}1/\theta}} \right) \cdot \left( \frac{1 + CV_{Y_0}^{\text{policy}2}}{1 + CV_{Y_0}^{\text{study}2}} \right)^{1-\theta} \frac{\rho(1+g_E^{\text{policy}})^{-1/\theta}}{\rho(1+g_E^{\text{study}})^{-1/\theta}} dE^{\text{policy}} \mu_{Y_0}^{\text{policy}1/\theta} (1 + CV_{Y_0}^{\text{policy}2})^{1-\theta}}{dE^{\text{study}} \mu_{Y_0}^{\text{study}1/\theta} (1 + CV_{Y_0}^{\text{study}2})^{1-\theta}}
\]

The transfer function for mean WTP elicited at study site and policy site as a single payment for a change in the level of the environmental good, \(\text{WTP}_{CPF,dE}(\mu_{Y_0}, CV_{Y_0})\) (Eq. (22)), is given as
The transfer function for mean WTP elicited at study site and policy site as a single payment for a change in the growth rate of the environmental good, \( \text{WTP}_{\text{CPF,}dE}(\mu_{Y_0}, CV_{Y_0}) \) (Eq. (24)), is given as

\[
\mathcal{T}_{\text{CPF,}dE}() = \frac{\text{WTP}_{\text{CPF,}dE}(\mu_{Y_0}^{\text{policy}}, CV_{Y_0}^{\text{policy}})}{\text{WTP}_{\text{CPF,}dE}(\mu_{Y_0}^{\text{study}}, CV_{Y_0}^{\text{study}})}
\]

\[
= \frac{1}{\alpha} \frac{1-\rho(1+g_E^{\text{policy}})^{\frac{\theta}{1-\theta}}}{1-\rho(1+g_E^{\text{policy}})^{\frac{\theta}{1-\theta}}} E_0 \frac{dE_{\text{policy}}}{\mu_Y^{\text{policy}}^{1/\theta}} \left(1 + CV_{Y_0}^{\text{policy}}^2\right)^{\frac{1-\theta}{2\theta}}
\]

\[
= \frac{1}{\alpha} \frac{1-\rho(1+g_Y^{\text{study}})^{\frac{\theta}{1-\theta}}}{1-\rho(1+g_Y^{\text{study}})^{\frac{\theta}{1-\theta}}} E_0 \frac{dE_{\text{study}}}{\mu_Y^{\text{study}}^{1/\theta}} \left(1 + CV_{Y_0}^{\text{study}}^2\right)^{\frac{1-\theta}{2\theta}}
\]

\[
\cdot \sum_{t=0}^{\infty} \left( \prod_{\tau=0}^{t} \delta^{\text{policy}} \right) \left(1 + g_Y^{\text{policy}}\right)^t
\]

\[
\cdot \sum_{t=0}^{\infty} \left( \prod_{\tau=0}^{t} \delta^{\text{study}} \right) \left(1 + g_Y^{\text{study}}\right)^t
\]

\[
= \left( \frac{E_0^{\text{policy}}}{E_0^{\text{study}}} \right)^{\frac{\theta}{1-\theta}} \frac{dE_{\text{policy}}}{dE_{\text{study}}} \frac{\rho(1+g_Y^{\text{policy}})^{\frac{\theta}{1-\theta}}}{\rho(1+g_Y^{\text{study}})^{\frac{\theta}{1-\theta}}} \left(1 - \rho(1+g_Y^{\text{study}})^{\frac{\theta}{1-\theta}}\right)^2 \left(1 - \rho(1+g_Y^{\text{policy}})^{\frac{\theta}{1-\theta}}\right)^2
\]

\[
\cdot \frac{\mu_Y^{\text{policy}}^{1/\theta}}{\mu_Y^{\text{study}}^{1/\theta}} \left(1 + CV_{Y_0}^{\text{policy}}^2\right)^{\frac{1-\theta}{2\theta}}
\]

\[
\cdot \frac{\mu_Y^{\text{study}}^{1/\theta}}{\mu_Y^{\text{policy}}^{1/\theta}} \left(1 + CV_{Y_0}^{\text{study}}^2\right)^{\frac{1-\theta}{2\theta}}
\]

\[
\cdot \sum_{t=0}^{\infty} \left( \prod_{\tau=0}^{t} \delta^{\text{policy}} \right) \left(1 + g_Y^{\text{policy}}\right)^t
\]

\[
\cdot \sum_{t=0}^{\infty} \left( \prod_{\tau=0}^{t} \delta^{\text{study}} \right) \left(1 + g_Y^{\text{study}}\right)^t
\]

\[
= \left( \frac{E_0^{\text{policy}}}{E_0^{\text{study}}} \right)^{\frac{\theta}{1-\theta}} \frac{dE_{\text{policy}}}{dE_{\text{study}}} \frac{\rho(1+g_Y^{\text{policy}})^{\frac{\theta}{1-\theta}}}{\rho(1+g_Y^{\text{study}})^{\frac{\theta}{1-\theta}}} \left(1 - \rho(1+g_Y^{\text{study}})^{\frac{\theta}{1-\theta}}\right)^2 \left(1 - \rho(1+g_Y^{\text{policy}})^{\frac{\theta}{1-\theta}}\right)^2
\]

\[
\cdot \left(1 + CV_{Y_0}^{\text{policy}}^2\right)^{\frac{1-\theta}{2\theta}} \cdot \left(1 + CV_{Y_0}^{\text{study}}^2\right)^{\frac{1-\theta}{2\theta}}
\]

\[
\cdot \sum_{t=0}^{\infty} \left( \prod_{\tau=0}^{t} \delta^{\text{policy}} \right) \left(1 + g_Y^{\text{policy}}\right)^t
\]

\[
\cdot \sum_{t=0}^{\infty} \left( \prod_{\tau=0}^{t} \delta^{\text{study}} \right) \left(1 + g_Y^{\text{study}}\right)^t
\]

\[
= \left( \frac{E_0^{\text{policy}}}{E_0^{\text{study}}} \right)^{\frac{\theta}{1-\theta}} \frac{dE_{\text{policy}}}{dE_{\text{study}}} \frac{\rho(1+g_Y^{\text{policy}})^{\frac{\theta}{1-\theta}}}{\rho(1+g_Y^{\text{study}})^{\frac{\theta}{1-\theta}}} \left(1 - \rho(1+g_Y^{\text{study}})^{\frac{\theta}{1-\theta}}\right)^2 \left(1 - \rho(1+g_Y^{\text{policy}})^{\frac{\theta}{1-\theta}}\right)^2
\]

\[
\cdot \left(1 + CV_{Y_0}^{\text{policy}}^2\right)^{\frac{1-\theta}{2\theta}} \cdot \left(1 + CV_{Y_0}^{\text{study}}^2\right)^{\frac{1-\theta}{2\theta}}
\]

\[
\cdot \sum_{t=0}^{\infty} \left( \prod_{\tau=0}^{t} \delta^{\text{policy}} \right) \left(1 + g_Y^{\text{policy}}\right)^t
\]

\[
\cdot \sum_{t=0}^{\infty} \left( \prod_{\tau=0}^{t} \delta^{\text{study}} \right) \left(1 + g_Y^{\text{study}}\right)^t
\]

\[
= \left( \frac{E_0^{\text{policy}}}{E_0^{\text{study}}} \right)^{\frac{\theta}{1-\theta}} \frac{dE_{\text{policy}}}{dE_{\text{study}}} \frac{\rho(1+g_Y^{\text{policy}})^{\frac{\theta}{1-\theta}}}{\rho(1+g_Y^{\text{study}})^{\frac{\theta}{1-\theta}}} \left(1 - \rho(1+g_Y^{\text{study}})^{\frac{\theta}{1-\theta}}\right)^2 \left(1 - \rho(1+g_Y^{\text{policy}})^{\frac{\theta}{1-\theta}}\right)^2
\]

\[
\cdot \left(1 + CV_{Y_0}^{\text{policy}}^2\right)^{\frac{1-\theta}{2\theta}} \cdot \left(1 + CV_{Y_0}^{\text{study}}^2\right)^{\frac{1-\theta}{2\theta}}
\]

\[
\cdot \sum_{t=0}^{\infty} \left( \prod_{\tau=0}^{t} \delta^{\text{policy}} \right) \left(1 + g_Y^{\text{policy}}\right)^t
\]

\[
\cdot \sum_{t=0}^{\infty} \left( \prod_{\tau=0}^{t} \delta^{\text{study}} \right) \left(1 + g_Y^{\text{study}}\right)^t
\]
References


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